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ABSTRACT

Abstracts and/or complete papers presented at this conference are provided in seven sections. Section 1 includes papers focusing on microcomputers and education. Section 2 includes a research review on spatial ability and its relation to mathematics education and two studies, focusing on spatial ability tasks for mathematics teachers/students and problem-solving for Yapese elementary schools. Papers in section 3 examine factors related to post year 10 mathematics. Studies in section 4 focus on first year university students' mathematical performance on the 1982 end of course examination, difficulties encountered in applying mathematical skills to solving engineering problems, and engineering mathematics. Section 5 includes a review of the Mathematics Education Centre (Papua New Guinea University of Technology--PNGUT) and studies on: abilities of senior secondary students at proportionality tasks, use of calculators in provincial high schools, and development of rational number concepts. Section 6 includes papers examining relationships between arithmetic and algebra, and the availability of mathematics resources in the Morobe Provincial High Schools. Five papers focusing on problem-solving at PNGUT are presented in section 7, including a literature review on the topic and an examination of how understanding the nature of a problem is helpful in developing successful problem solvers. (JN)

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RESEARCH IN MATHEMATICS EDUCATION IN PAPUA NEW GUINEA

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RESEARCH IN MATHEMATICS EDUCATION

IN

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1983

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Instructional Computing-

Microcomputer Capabilities for Computer Presented Lessons.

by

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Papua New Guinea

It is an event as profound as the First Wave of change unleashed ten thousand years ago by the invention of agriculture, or the earth shaking Second Wave of change touched off by the industrial revolution. We are the children of the next Transformation, the Third Wave.

(Toffler p. 25)

NCTM "recommends that...mathematics programs take full advantage of the power of...computers at all grade levels."

(NCTM, page 1)

Projects that involve students and teachers in the use of computers in instruction at the tertiary and secondary school level are not new. Projects such as PLATO, TICCIT, and the Computing Lab. of Alfred Bork at U.C. Irvine are well known. They involved the use of large mainframe computers and terminals for student use. With the development of the microprocessor in the early 1970's, followed by the development of affordable and easy to use microcomputers in the late 1970's, has come the motivation to fit these devices into education at all levels and in all contexts. I think we really are on the Third Wave and in a very short time there will be a change in the way we think about schools, education and what they can be.

In the past, the use of new technologies has had a less than revolutionary effect on education. The slow up-take is partially due to the lack of knowledge about the uses and capabilities of the technology and the lack of software that can be used in an educationally meaningful way. Film, audio and video technologies all have these problems. The evolution of teacher change and software change is very slow. It is interesting to think about the software changes that have occurred recently. These have been due to the use of computer technology in film production. Many of the visual features of films are now images that have been enhanced by computers.

The introduction of computers into education has also been slowed due to the lack of expertise and software. But the evolution of this technology seems to be very much more rapid than those mentioned above. It is said that the computer is a tool of the mind. It helps us to do mental work more rapidly. This is certainly true. A more powerful feature of the computer is that it helps us to create its own software. It helps us to put ideas into software form more rapidly than we could with other technologies. It helps us to enhance the ideas with sound, animation and color more rapidly than other technologies. It looks like the computer itself is going to have a major role in speeding up the evolution of the software that will be used on it in education. It just can't wait to show us all it is capable of with a little imput of imagination from the teacher.

The changes that have taken place in microcomputer software in the last two years have been revolutionary. The revolution is only a year old in the public's eyes, but I am assuming that the changes were occurring amoung small groups of hockers before they went public. The revolution is just not touching education.

Let's go back a few years. Education software for microcomputers in the late 1970's attempted to use all the capabilities of micros that were known to mankind.

Early software used the micro's capability to be a patient tutor and drill instructor. It allowed some interaction with the activity of the micro, and allowed self-pacing of presentations to the student. The software got the micro to respond to learner/user input in a reinforcing way to encourage the learner/user to continue to use the micro. The later was accomplished with sound and graphics features. Some thought was put into the presentation of the materials, but the presentations tended to be similar to that of earlier technology with which the authors were all fimiliar, printing.

This has changed! The source of change was not the needs of education, but the great thirst people have for entertainment. There was money to be made in developing the capabilities of micros that could be used to develop games for people. Some of the change also came from the need to make computers easy to use, especially in business. Some of the change,I expect, also came from individuals interested in extending the capabilities of the micro just to see what it could do, moving the edge of knowledge as a researcher does without invisioning a use or need for that knowledge. The challenge is enough.Just as the need of national security and the quest for space travel have generated knowledge and products that effect our daily lives, education will be effected by the knowledge and skills developed to satisfy other needs.

The skills and knowledge developed for business and gaming can now be applied to the production of educational software to help learners and teachers to achieve their educational goals. The "computer screen" presentations in a textbook format a few years ago are beginning to be replaced by action packed scenes that could help students to develop ideas and skills better than other means. This is just beginning. Teachers are generally involved with the development of texts or the authors were teachers in the past. This doesn't seem to be so with software for microcomputer use. Much of the early software was a weak attempt to enter the educational marketplace with a half-baked product to test the market. The market wasn't ready and the product wasn't acceptable. Teachers are beginning to be interested and involved and software is being produced that will help teachers succeed with their educational goals. The development of software for educational purposes needs to integrate teachers' intuitive ideas, accepted educational ideas and the present goals of educational institutions to be accepted and effective. The universities research institutes need to look into software development as a research area and look at what software <u>can be</u> if all the micro's capabilities are extended to their ? limits?. They need to look at what education <u>can be</u> with the introduction of microcomputers and other computer enhanced technologies.

The main part of this session will be used to display some of the capabilities of micros as expressed in some educational software and in entertainment software. Many examples have to be drawn from the entertainment area because the motivation is not yet there for publishers/authors to put a lot of energy into educationally sound software. This may be where the Universities and Teachers Colleges could come in,by directing some resources into computer software development work. Some activity is going on already.

It is worthwhile to take a brief look at uses of computers in education to see where the capabilities of micros fit in. Student programming of computers has been an acceptable activity for years. As micros develop, they will be able to use higher level languages that are easier for students to learn. The use of computers to do administration and lesson development tasks could become a valued capability (an offshoot of business needs). Perhaps dedicated microprocessor controlled devices will appear for test evaluation and students' assessments and record keeping with voice imput, of course! The broadest area of use is that of presenting learning activities via the micro's video screen. This is the usage on which our deliberations should center today. It is the area open to the greatest change and greatest potential pay off to education. Within each of these three catagories of microcomputer usage, there are many discriminations that will be useful when working on the production of software for micros for use in education. I leave you with the question, What can education be like with the microcomputer?

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TOFFLER, A. The Third Wave, New York: William Morrow & Co. 1980

NCTM, Agenda for Action, Reston, Virginia: NCTM, 1980

M.E.C. CONFERENCE TEST

NAME

1)
FIND THE PATTERN AND FILL IN THE BLANKS.

68, 67, 65, 62, 58, ____, ____

2)
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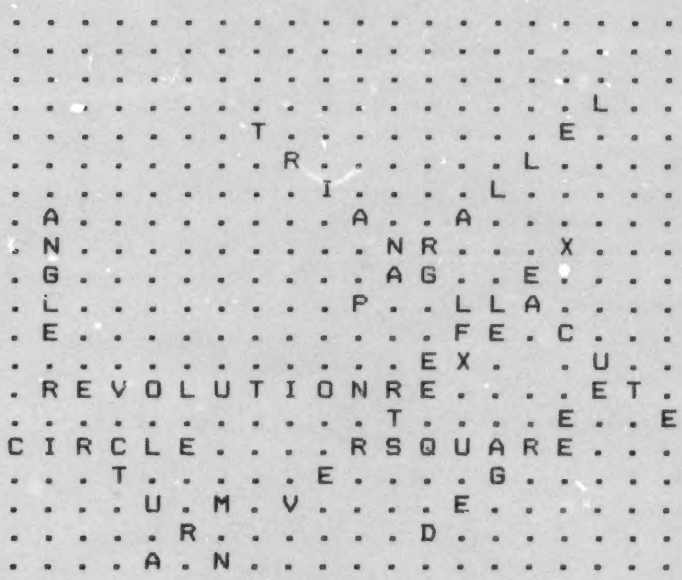
SHAPE AND ANGLE FUN



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SHAPE AND ANGLE FUN

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MICRO-COMPUTERS IN EDUCATION - SURELY NOT JUST A ROTE PRACTICE AND AUDIO-VISUAL MACHINE?

C. WILKINS
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ABSTRACT

With the widespread availability of micro-computers every programmer and quasi-programmer has added to the countless number of computer instruction, rote practice or remediation programs available for students. Some are good! However attempts have been made to use the new technology more imaginatively - with the computer being the nucleus of a discovery style of extra curricular instruction. The paper both questions the potential of computer assisted instruction in education and outlines the benefits of a free style student/computer education.

INTRODUCTION

Two other speakers at this conference, Mr Rod Selden of the University of Technology and Mr Harold Frederick of Goroka Teachers College, are, I know, keen advocates of the use of micro-computer software and are optimistic about its potential. I can not share their full enthusiasm.

During the last three years I have, on occasions, used a micro-computer in the classroom and for personal enjoyment. During that period I have at times become enthused with its possibilities in education and depressed with the limitations placed upon it caused by the non-availability of useful software packages. In this paper I will detail what I see to be the present limitations inherent in "Computer Assisted Instruction". Because otherwise this paper would be hopelessly pessimistic, I will suggest one possible way in which, in future, the micro-computer might be more than usefully employed in the classroom.

LIMITATIONS ON COMPUTER ASSISTED INSTRUCTION

Let me set my terms of reference by mentioning a little tried but widely critisized use of computers in education - Computer Based Instruction. This is a system that manages student education. The machine is the teacher, giving individualized instruction (the appeal of the system), diagnosing incorrect responses, providing remediation, and assessing the progress of students. Perhaps this idea has reached its zenith with the PLATO system - an extremely flexible and sophisticated software package based on a large mainframe computer system. PLATO has good monochromatic graphics, an easy to use authoring system, provision for student interaction via keyboard and/or screen, and has programs developed and tested by educators. However, although it would seem to be a very impressive system, it has generally been used to its full potential only for short post secondary teaching projects.

Any lesser aim of software or machine is generally labelled as Computer Assisted Instruction (or CAI, to use the jargon). This includes simple demonstration programs, rote learning packages, a compilation of graded questions (with perhaps a score counter or record keeper), and the like. PLATO, as used in schools today, is generally only utilized to the CAI level.

When an instructor decides to use some CAI material, he/she has two paths to follow - to search for professional

material suitable for his/her needs, or to write software. Both are less than satisfactory.

The first course is full of hazzards. There is an incredible amount of software presently available, and the amount available is increasing at an ever increasing rate. However I, at least, am less than enthusiastic about this wealth of material. Why? The following reasons come readily to mind:

- it is difficult to locate (here in PNG at least). There are magazines (e.g. Educational Computing) which list a variety of available packages but supply little other useful information.

- the reviews that do come to surface can be misleading. The user can not have great confidence that a package will live up to its reviews, or that the package will serve his purposes exactly. There is only one way to find out - buy! And this can soon be an expensive exercise.

- programs tend not to be machine transportable. In several years the user will no doubt find the need to update his micro with a state of the art machine. Unless he specifically adapts the machine to mimic the earlier version, old software packages will become obsolete.

- the quality of software is so incredibly varied. A lot of material masquerading as professional is written by (and obviously written by) amateurs. In some instances large sums of money are being asked for what might not unfairly be termed well documented trash.

The above comments probably apply to software in all fields (business applications, engineering, architecture, etc) but are probably especially relevent to education. The packages in education should be written by professionals, but if we attempt to define "professional" in this context I doubt that many programs could claim a pedigree background.

What qualities should a professional educational software producer have? I would suggest ideally three, but three that may be incompatible for a single writer.

The programmer should have flair - the ability to make the package stimulating, interesting and absorbing. It is a quality hard to define, but is certainly evident (in the context of computer games packages) in the video game 'Pacman'. This simple maze game uses less visual and audio gimics than many of its 'Space Invader' relatives, but has incredible appeal. I have yet to see many educational packages that have this spark (which is not to deny that they

exist).

Secondly, the software writer should be an educational and child psychologist. Questions that need to be addressed include: "how can graphics, etc, be used to the best advantage with a target age group?", "what format (e.g. game) would best aid the transfer of the mathematical concept to a user?", "what degree of user/machine interaction should there be?", and "how many and what types of reward should be offered?"

Finally a quality that may be incompatible with the last. The writer of these educational packages should be a computer 'freak'. This is not intended as a barbed insult but to indicate the need for the programmer to have an intimate knowledge of his medium. Only in this way will the programs become slick. Too many programs available are written BASIC, interpretted line by line, and are hence frustratingly slow.

I daresay that extremely good "educational" software packages will become available in the not too distant future (especially if they are written by a team combination of educator and computer expert), and that word of the better ones will be spread between users. However programs, as they become more sophisticated (as I urged above), will invariably be more machine dependent - with again the problem that a new machine will mean a new library of software. Or can we expect standardization in the near future? I hope not, because that would mean an end to the evolution of an intriguing piece of equipment.

DO-IT-YOURSELF PROGRAMMING

I would guess that most educators who have invested in, or who have been lucky enough to have the unlimited use of, a micro-computer have indulged in the do-it-yourself instructional program - possibly a demonstration program. I would also guess that in the majority of cases the exercise in program writing has been far more beneficial to the education of the programmer than his program ever will be to his intended users! But this in itself is insufficient reason to critisize the efforts of the programmer.

When the amateur writes a program he/she will use interpretive BASIC, or perhaps sometimes a compiled language such as PASCAL, and possibly also incorporate some machine subroutines. This forces any program that uses, say, complex graphics to be slow. In addition invariably anything that the do-it-yourselfer does will have been done hundreds of times before and, at least sometimes, better. If the purpose of the software writing exercise is to prepare an educational

package, then this is an incredible waste of man power. A non-trivial project will take many hours of work before the program reaches the stage that its execution results in an output aesthetically acceptable to the programmer, and for all the obvious programming errors to be removed.

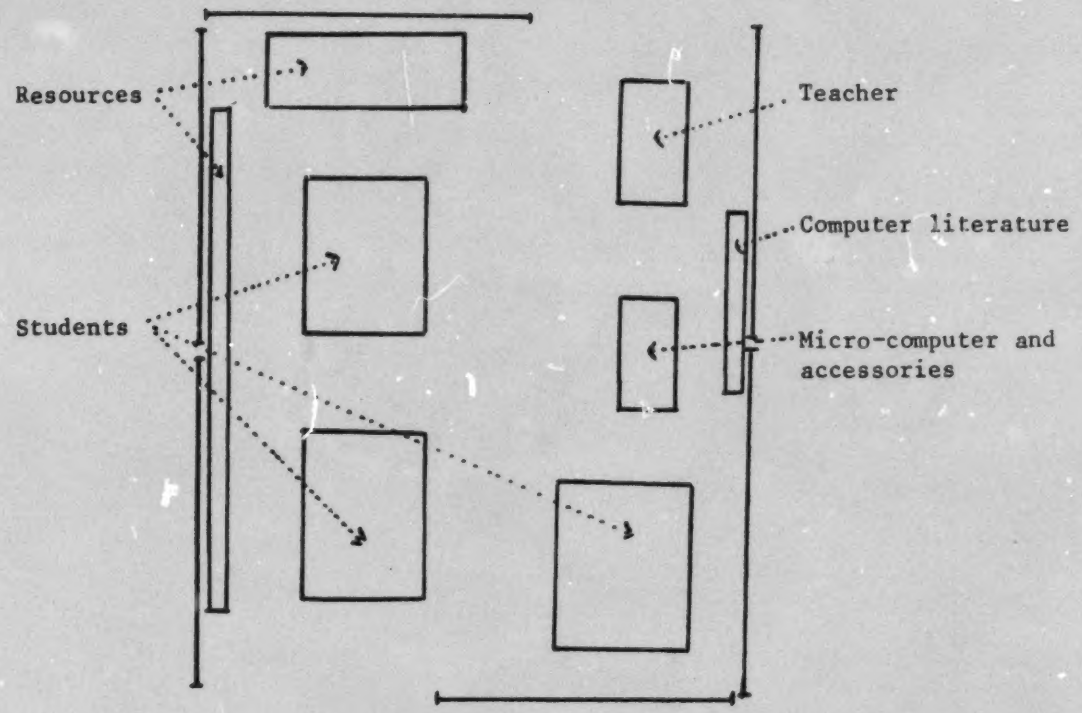
One problem with the do-it-yourself programs is that they often do have less than adequate documentation - in particular there tends not to be an accompanying users guide for the package. As a result these programs are far less user transportable than are professional packages with their large (and sometimes complicated) instruction manuals. There are two advantages of writing your own programs, though. They are an aweful lot of fun to write, and the enthusiasm developed by the writer may rub off onto the students for whom the software was written - irrespective of its quality.

These observations on professional and amateur material lead me to the conclusion that if a micro-computer is to be used in education through the medium of computer assisted instruction, then it will be just one other item in a bag of tricks (which include games, films and other audio-visual aids) used to add variety to the students' instruction. But I see, in another context, that the micro-computer can play a significant role in education, and in mathematics education in particular...

A POSSIBLE MODEL FOR THE FUTURE

The following observations stem from a brief visit I made to one school in the United States some 18 months ago. There Dr. Jan Flake of the Florida State University (Tallahassee, Florida) was monitoring the progress of a group of grade 6 primary school children provided with an Apple II micro-computer, disk drive and colour monitor. I would refer any interested persons to contact Dr. Flake for details and progress of this project. Here I will briefly outline the form of student/computer interaction.

The class was a group of 15 exceptional (good) students, the school was open space, and the area allocated to these students centered on their micro-computer. The diagram on the next page illustrates the room design.



The students were allowed to interact with the computer in anyway they liked - I will refer to this as a free-form instruction. The only constraint was that they must work in groups (a very worthwhile constraint, given the widespread concern about creating a future generation of computer zombies!). The children decided how to use the computer, what games they wished to play, what programs (if any) they wanted to write and what literature (mostly computer magazines such as Applecart) and software packages they wished to purchase within a limited budget. Their array of previously purchased materials was impressive, but their knowledge of the machine incredible (they had been using the machine for, I think from memory, just over a year). The beauty of the situation was that the machine was used as a reward for early completion of prescribed classroom work, and that when the students were using the micro-computer in an unconstrained manner the machine presented its own rewards. It remains to be seen whether this association will affect, one way or other, the students' mathematical development. I believe that it will. This type of activity must reinforce two important aspects of mathematical development - logic and enquiry.

That the micro-computer can be a good developer of logic there can be little doubt. Unlike pencil and paper mathematics problems an error in logic is detected

immediately. as the machine performs erratically when executing the instructions. All too often a student can be oblivious to a logical error when aswering a question. and corrections by the teacher can fall upon deaf ears. However when using. say, graphics on a micro-computer and a set of instructions do not yield the desired result the student will begin to enquire why. The exercises in logic can be at a very elmentary level. such as moving an object around the screen with a Turtle graphics type program, or. at the other end of the scale. coding a complicated program.

Also on enquiry. I find that most people, when presented with a software package written by someone else with more experience. will be impressed with certain new techniques. and will attempt to find the secrets of these techniques. They may do this by asking. or even attempt to find the secret by trying to understand a small piece of computer coding. Given that we as mathematics educators would aim to make students inquire into unknown or poorly understood areas of knowledge. such training with a computer must prove useful.

I suggested earlier that a do-it-yourself programmer might get more benefit from writing a piece of software than the students who finally used it. For the same reason I would expect that a free-form interaction of student with machine would be much more beneficial to the mathematical development of the student than is access to a pre-written educational software package. Micro-computers are passive machines. Unless the package has flair. once the sheer thrill of a new gadget has worn off it would be all to easy for the student to sit before the keyboard and dreamily press buttons while the package proceeds on its merry way assuming the full attention and interest of the user.

IN CONCLUSION

But this is the real world. The American class I described above was of exceptional children. It was small. It was in an affluent part of an affluent country. Papua New Guinea schoolrooms are nearer the other end of the spectrum. Even if micro-computers continue to get relatively less expensive it will be a long time before small groups of students will have access to a machine full time. It does. however. perhaps give us somewhere to aim if. as I think may happen. the present euphoria with the potential of computer packages fizzles as has the use of other audio-visual aids before.

THE USE OF COMPUTERS FOR CONCEPT DEVELOPMENT

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The purpose of this paper is to describe some current work in developing learning experiences utilising the capabilities of the computer.

This work is being carried out in connection with two courses (MA 139 and MA 392) at the Papua New Guinea University of Technology. Both courses cover fundamental work on functions (polynomial, trigonometric, exponential and logarithmic) and calculus (differentiation and integration).

Graphical representation and interpretation is an important aspect of these courses. Accordingly, the emphasis so far has been on the use of the graphics capabilities of micro-computers to construct graphs.

It has only been possible to do this work since sufficient computers have become readily available for students to use. We are currently working with 10 Apple II microcomputers, situated in one room. The layout of the room is such that 2 or 3 persons can be comfortably seated at each terminal.

A large number of graphing programs have been written for the Apple II. Of those that I have seen, most have been unsuitable for our purposes, being either too specialised in some areas, or too business oriented. The most useful program so far has been DISCOVERY LEARNING IN TRIGONOMETRY (Kelly, 1982). I am currently developing a package entitled INVESTIGATIONS IN FUNCTION GRAPHING which is being specifically designed for use in the courses mentioned above. It is being modelled on the successful features of the DISCOVERY LEARNING IN TRIGONOMETRY program, and is intended to facilitate the investigation of a wide class of functions. These two programs will be discussed in more detail later in this paper.

I am still looking for suitable programs and would welcome any assistance that others may offer.

PHILOSOPHY OF MATHEMATICS EDUCATION

Because the computer is such a versatile tool, there are many ways in which it can be used in education, some of them good, some not so good, and some clearly disastrous. It is important therefore to be clear on the philosophy underlying the use of the computer. This will be dependent on one's philosophy of mathematics education in general and so I will outline briefly the philosophy underlying the present work.

One can distinguish a number of important areas of mathematics learning, including skills, concepts, attitude, and the higher order skills of problem solving, logical thinking, analysis and creativity.

Everyone recognises the importance of all these areas, and everyone would emphasise them in their courses if it were easy to do so, but in practice most of what we ask our students to do amounts to little more than the performance of more or less straight forward skills in routine contexts. We are reluctant to ask students to do anything which depends critically on newly encountered concepts because we are not prepared to cater for the difficulty we know they will have. We are reluctant to spend time on activities which are specifically designed to promote concept development, because we feel that such activities will be time consuming and unproductive in terms of immediate improvement in skills. Because we baulk at spending time on concept development, our treatment of higher order skills, which cannot function without a suitable conceptual base on which to operate, is generally quite feeble.

This approach represents a very pessimistic philosophy of mathematics education but I am continually amazed at the widespread acceptance of this philosophy and the attempts to justify it. To be fair however I have to admit that I too often

lapse into this philosophy. It is an easy philosophy to implement and because it is compatible with what students and colleagues expect it is a safe philosophy, even if it is inadequate.

The current work is based on an alternate philosophy which recognises firstly the importance of each of the above mentioned areas of mathematics learning in its own right, and secondly the importance of the interactions that can take place between these areas. For example, emphasis should be given to concept development partly because it doesn't make sense to teach people methods without teaching them something about what the method does, when it should be used, how it works and so on, and partly because well developed concepts can assist in skill development (by organising and simplifying skills, by relating them to other things so they are easier to remember, and by providing a sensible basis for constructing new skills), and can provide a basis for the use and development of higher order skills. Likewise problem solving should be emphasised partly because it is an important ability in its own right, and also because it utilises, develops and reinforces skills, concepts and other higher order skills. Having recognised the importance of all areas of mathematics learning, and of the interactions between them, the alternate philosophy embodies a commitment to find ways to give emphasis to all areas.

PHILOSOPHY OF COMPUTER USAGE

In line with the above philosophy of mathematics education our philosophy of computer usage begins with the desire to find ways of emphasising concepts and higher order skills.

This philosophy differs from some of the early uses of computers as instructional aids. In these the program tried to provide a comprehensive management of the learning situation and what was essentially a sequence of programmed instruction. This sort of program mainly emphasised skills, which is the one area of mathematics learning which already receives

sufficient emphasis without the use of computers. The learning of some concepts and some higher order skills in a programmed learning context is possible, but such programs are the most difficult to write, and even with the best software available the options are limited. In the area of skill learning through programmed instruction, computers have proved to be effective at least in a narrow context, but, besides being a very expensive way to use computers, it has consistently produced a negative impact on attitude.

The very limited success of programmed instruction with computers should not be taken as an indication of overall limits to their usefulness. Computers are versatile and powerful tools. Instead of using the power to supply instructional sequences, a different and much more exciting approach arises out of the idea of putting this power in the hands of the learner. Let the computer be a tool not of the teacher but of the student. This idea opens up a whole new range of possibilities. The most significant aspect of this for us is that it shifts the emphasis from basic skill development to the development of concepts and higher order skills.

There is enormous scope for the investigation of the possibilities of using computers in this way. Although we need to proceed with caution I think there are good reasons for being optimistic about the possible benefits. In the future I hope to investigate specifically the possibility of using computers to develop logical thinking skills. The current work however has been aimed at concept development, although its actual effects in this and other areas have yet to be ascertained.

A PARADIGM OF COMPUTER USAGE

We can see in our work the following paradigm. The computer is programmed to carry out some task for which it is particularly suited, and which couldn't be done in other ways. This task has a product which is relevant to some form of mathematical analysis. The program is put in the hands of the student. He

controls the inputs, he receives the product and is required to interpret it. He can vary the inputs and investigate the variation in the product. He can investigate, as far as is appropriate, how the program itself works, and a concrete basis is provided for this activity. Every activity is open ended and there is opportunity for exploration and perhaps creativity. Even so, activities can be prescribed which focus on particular ideas or situations. The learning tasks are defined not by the program but by the teacher and the student.

This may seem abstract so I will give two examples. In the current work the program essentially draws graphs. Graphs are hard to draw accurately and it takes time. The computer can be made to draw them quickly and well. Instead of graphs being simply a chore they now become an object of investigation. Many graphs can be drawn, of different types and on different scales, and their features can be noted and related to features of the functions underlying them. Although it is early days yet the activities that have resulted from using the graphing programs have been interesting and promising. Many of these activities would have been difficult if not impossible without the computer. Some examples are presented in the next section.

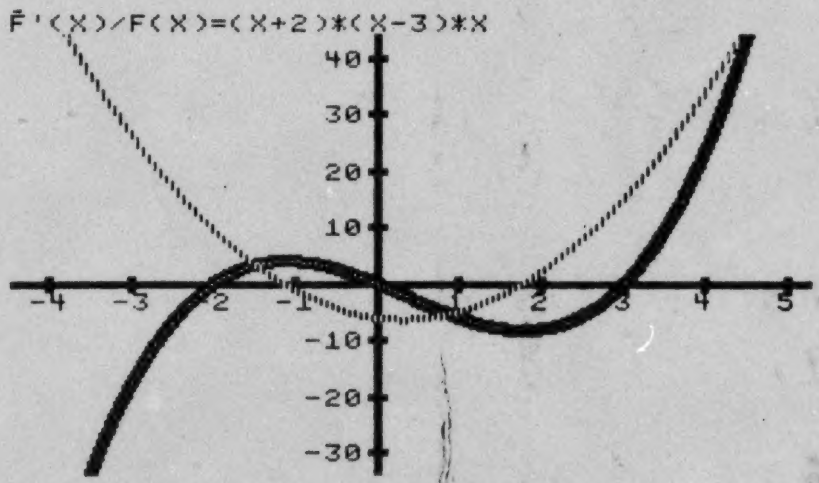
The second example in which the above paradigm is being followed is in a statistics course taught by Peter Sullivan here at the Papua New Guinea University of Technology. In this case the program is a statistics package. It does many statistical analyses quickly and effectively. Students can use the package and obtain results which they can then interpret. Without the computer individual analyses are time consuming in the extreme. Now it becomes possible to do many analyses, which can be used to investigate various effects and to get an intuitive feeling for statistical concepts. These ideas can then be related to the basic formulae and as far as seems appropriate the formulae themselves and the operation of the package can be investigated. As a specific example of this approach we might consider concepts related to the distribution of sample means. This

is typically treated as an abstraction but a good text might have one example presented in full. Statistics packages can take large numbers of samples, calculate population means, sample means, and statistics based on the sample means very quickly, thus providing concrete examples of these difficult concepts. Moreover these examples can be generated by the student himself, possibly even on his own data. The details of the formula, for example, of the variance of the sample means can be checked empirically.

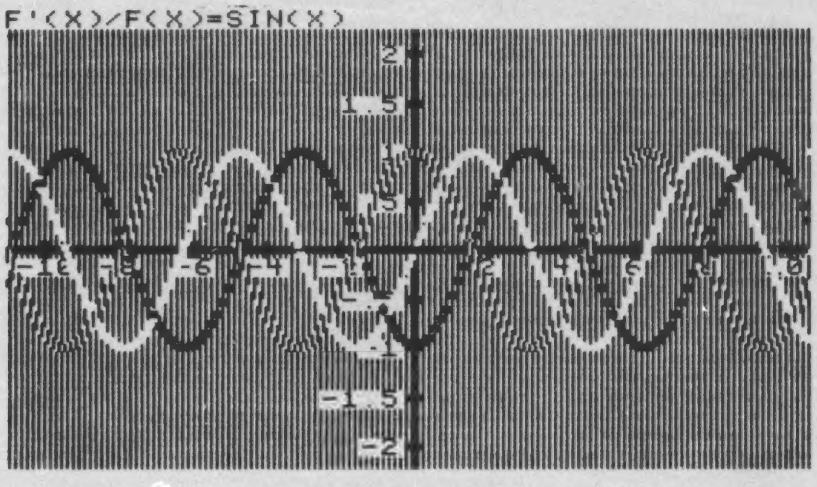
THE GRAPHING PROGRAMS

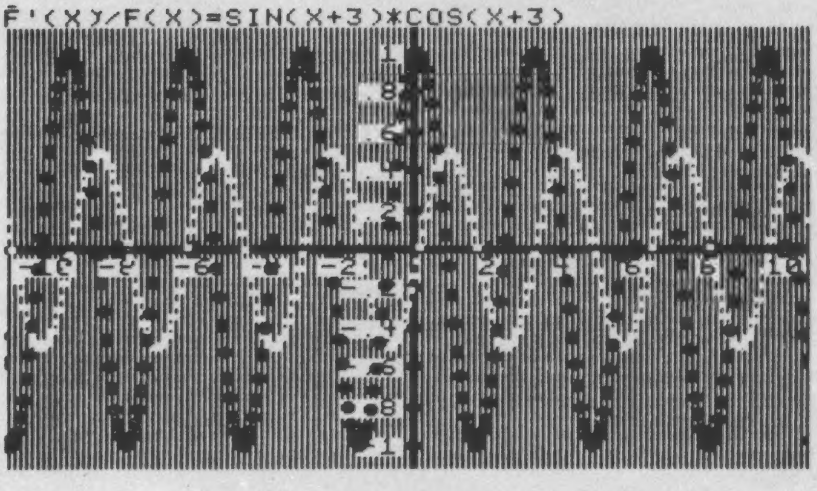
I wish to give some examples here of what I hope to achieve using the graphing programs, and to present some illustrations of some of the things that have already happened.

The most important objective is that the student should make connections between different mathematical ideas, and between different representations of the same idea. I want them to do this in the context of an activity specifically aimed at a given relationship, but I also expect that they will do it spontaneously in most of the activities they do. A calculus program, for example, draws gradient functions, second derivatives, integrals, and tangent lines. The resulting graphs can be interpreted in terms of positive and negative values, positive and negative slopes, zeros and turning points, points of inflection and so on. Such interpretations require accurate graphs and this the computer provides.

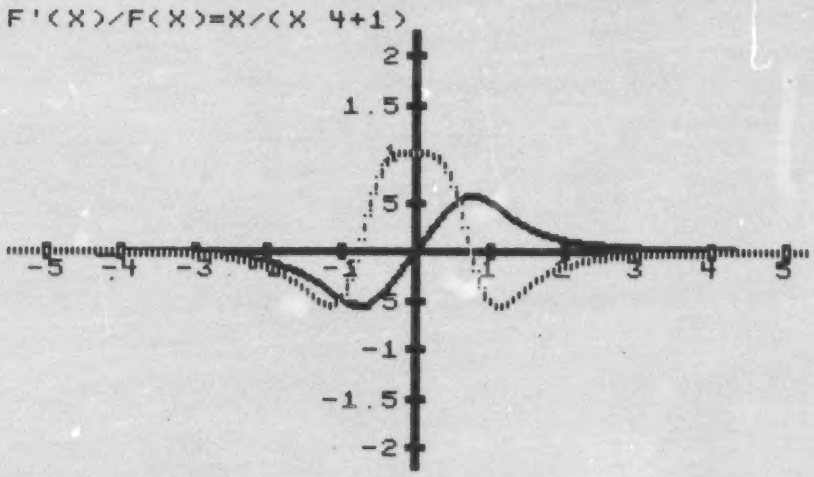


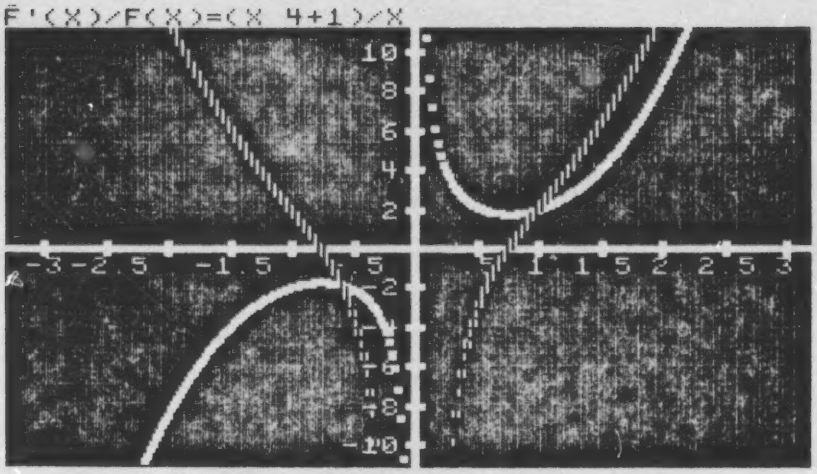
In the process of investigating these features of different graphs the student may chance upon, or be directed towards, sine graphs. He may then discover that the derivative of a sine graph still looks like a sine graph. This provides an experience of a relationship which might otherwise have only been treated analytically. Two of my students tried multiplying a sine function with a cosine function, as illustrated below, and they found that the result was a sine function. This will give us something to talk about when we do trig identities in a week or two. I suggested the students investigate the periods but they found it very difficult. This tells me, and them, something about their knowledge of periodicity.





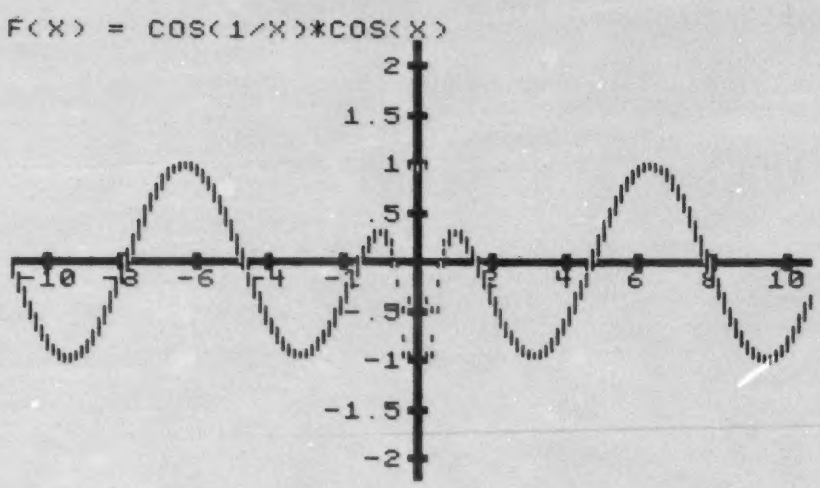
While carrying out this activity two of my students investigated the function illustrated below. They were meant to be interpreting gradients but they didn't fail to notice that they had discovered some odd and even functions. This topic had been introduced to them for the first time the day before but no suggestion had been made that they should look for odd and even functions in this activity. Would they have known that this function was odd if they hadn't seen the graph? What about the curious fact that the derivative of the odd function is an even function? This just happens to be the subject of a question in their next assignment. The students had great pleasure in evaluating the function and showing me that it obeyed the odd function property, and they did this without any suggestion from me that they should do so. I came back to them a little while later and found that they had taken the reciprocal of their original function and discovered two more odd and even functions. The derivative of the new odd function is still an even function. I wonder if they thought about the possibility that the reciprocal of any odd function is also an odd function?

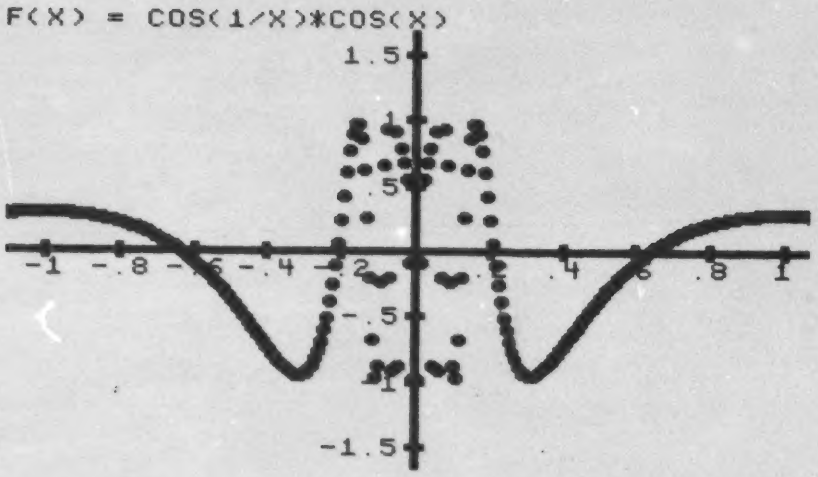




The computer will only draw a graph to a design specified by the student. It is easy for the student to design his graph by trial and error, which is important in the early stages and for new and difficult functions. However designing a graph for a specific purpose will draw on a variety of skills and concepts. Specifically it may draw attention to scale and scale related properties (e.g. periodicity in sine functions), to the magnitude of various functions (c.f. sine functions and cubics) and to the existence and location of critical points. The student can be given a challenge to get a good graph the first time, which requires him to work out these things in advance. But if he doesn't succeed rubbing out and starting again is easy, and as a side bonus he gets the wrong graph drawn in full which just may throw up something else of interest.

Two of the students tried multiplying cos(1/x) with cos(x), a function that I would not have dreamed of giving them. It looked like a nice graph but something funny seemed to be happening near the origin. So we changed the scale. I don't think they understood fully what was going on here, but notice that this intriguing function arose out of something they had initiated. They were interested in it because they showed it to me, but I wonder if they would have cared two hoots if I had shown it to them.





The use of the graphing programs enables the students to see many more graphs than they normally would, and to see them in the context of a meaningful activity. The behaviour of complex functions can be seen and, perhaps, interpreted. Unusual behaviour is not a point of difficulty, but rather a source of interest as it should be. Because the computer

draws accurately, the graphs they do see all have the right shape. How often do students see a well drawn quadratic or cubic graph? I am hopeful that merely seeing the correct shape will improve graphing skills.

The computer can expose misconceptions in an impelling and constructive way. I am repeatedly emphasising to my students that, for example, sin(x) does not mean two numbers multiplied together. This is a very hard message to get across. To input functions into a computer it is necessary to insert all multiplication signs. I was not particularly surprised to find that over half the class, working in pairs, inserted a multiplication sign between sin and (x). The program didn't work until this was removed. This was a very revealing experience for the students and I think they have got the message at last.

So far the students' attitudes to this work have been encouraging. They know they are doing something unusual but they do not seem to think they are wasting their time. On the contrary I am getting the impression that they recognise the importance of this work. I do not often run hour long classes which go 10 minutes overtime without anyone noticing. The problem is not keeping their attention but getting rid of them at the end.

THE TRAINING OF SPATIAL ABILITIES AND ITS RELATION TO MATHEMATICS EDUCATION: A REVIEW

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ABSTRACT

Training studies of a variety of spatial abilities which have some relevance to mathematics education are reviewed. The studies are classified according to the nature of the spatial ability being trained using a distinction derived from two ability constructs recently introduced by Bishop (1980b). An evaluation of the studies indicates that those which attempt to train the ability to interpret figural information have been largely successful whilst for those which attempt to train the ability to process information visually the available evidence is either inconclusive or suggests that they have not, on the whole, met with success.

SPATIAL ABILITY AND MATHEMATICS

A mathematics educator's interest in spatial training programmes is justified by the extent to which the performance on school mathematics tasks depends on the pupils possessing a wide variety of spatial abilities. During the process of learning mathematics, from primary to tertiary level, a whole spectrum of spatial abilities is invoked and these range from the ability to read and interpret diagrammatic information through to the ability to carry out complex visual processing of information. From the time a child enters school, much of his mathematics learning depends upon his ability to process information presented in visual terms, e.g. the teacher's blackboard work; diagrams, drawings or photographs that appear in textbooks. As he proceeds through school the demands made upon his spatial abilities become greater either because the material being presented becomes more abstract, as with such topics as the number line, graphs, geometry and analytical geometry, or because of the increasing degree of complexity of mental visual processing required to solve certain problems, e.g. the mental translation or rotation of 2–D figures in transformation geometry, or the visualization of 3–D solids or surfaces in vector algebra, 3–D geometry, or integration.

A number of spatial training programmes discussed in this review have been motivated by reasons other than the desire to improve the sort of spatial abilities required to deal with the visual aspects of mathematics mentioned above. The rationale for these training programmes proceeds as follows: from the psychometric factorial

studies of mathematical ability, spatial abilities consistently appear is an important underlying component. This view, for example, is held by Macfarlane Smith (1964) who regards spatial ability as the key component of mathematical ability. More recently, from the problem-solving literature, it is apparent that many subjects use visual imagery and visual processing of information in their attempts to solve problems, a view confirmed by some eminent mathematicians, such as Jacques Hadamard, who report that visual imagery plays a crucial role in the solution of problems. Thus, if a suitable training programme were able to improve a subject's spatial abilities one would expect a corresponding beneficial spin-off in terms of his mathematical performance.

This review, then, will take a wide view of those abilities which may be termed spatial and which have some relevance to mathematics education. In the literature there is by no means any generally accepted definition of the term 'spatial ability'. Differing views of spatial ability derive from the various schools of psychology: the psychometric school, the developmental psychologists, and the information processing school, each with its own theoretical viewpoint and research methodology. As some of the studies to be discussed here are based on either a psychometric or a developmental view of spatial ability, these views will discussed first before proceeding to establish a theoretical vantagepoint from which the spatial training studies can be reviewed.

Spatial ability: the psychometric view

The psychometric view of the spatial ability construct has been complicated by the use of a variety of factor analytic methods, on the one hand, and by disagreement among factor analysts as to the number and nature of spatial factors on the other. British factor analysts have predominantly used factoring techniques based on a hierarchical model of abilities with Spearman's 'g'

at the apex of the hierarchy and spatial ability as a secondary factor lying below 'g'. This view (see, for example, Smith, 1964; Holzinger and Harman, 1938; Burt, 1949; Vernon, 1950) regards spatial ability as a unitary construct with the implication of it being both a fixed and hereditary ability, and this is likely to account for the small number of British studies which attempt to modify spatial ability by training. The American view, on the other hand, is that there is a number of quite different space factors. There have been many attempts to adequately define them (see, for example, Thurstone, 1938; French, 1951; Michael et al., 1957; French, Ekstrom, and Price, 1963). That little unanimity has been reached regarding the number and nature of the spatial factors is clear when we compare the conclusions of recent major reviews of the literature. McGee (1979) concludes that there are two factors, Lohman (1979) three; both are agreed, however, that Spatial Orientation (S—O) and Spatial Visualization (Vz) are two distinct and important factors, whilst not agreeing as to how they are defined.

This disagreement regarding the definition of spatial factors needs to be borne in mind when considering spatial training studies carried out in the psychometric tradition. Many of these studies have used tests which purport to measure the S—O and Vz factors; a lack of clarity in how these factors are defined is likely to lead to a corresponding lack of clarity regarding how they might best be trained. Further, there is evidence which indicates that many of the test instruments designed to measure these factors do not in fact do so. The tasks which appear in a particular spatial test may be solved in several different ways, an S—O spatial task being solved, for example, using a typical Vz strategy. Barratt (1953), Myers (1958b), and French (1965), provide evidence of the variety of strategies subjects use in the solution of spatial tasks. Guay, McDaniel and Angelo (1978)

note that two main types of processing strategy tend to be used: gestalt and analytic. Gestalt processing, which Smith (1964) regards as being the essence of spatial ability, involves the construction and transformation of mental images as organized wholes; the analytic strategy involves trial and error checking of relationships between or within various figures and may involve logical and 'non-spatial' reasoning. Lean (1981), using a sample of English secondary school students, investigated by questionnaire their strategies used in the solution of spatial tasks of the type found in six standard psychometric spatial tests. Strategies were classified as either gestalt or analytic. The results indicated that for all six spatial item types subjects used more than one typical solution strategy and that for five item types solution strategies of both an analytic or a gestalt nature were reported. Upper ability subjects tended to be more analytic in their approach. A similar solutions strategy questionnaire was developed by Burden and Coulson (1982) who classified strategies using a visual-verbal dichotomy and a whole-part dichotomy (i.e. whether subjects dealt with the item stimulus as a whole, a gestalt approach, or in parts). On most of the spatial tests used in this study subjects exhibited a range of strategies including those of a verbal, analytic nature. Thus the factor reference tests which are typically used in the psychometric spatial training studies are likely to reflect non-spatial, analytic, abilities as well as spatial abilities. Viewed another way, a spatial training program, which has been claimed as successful on the basis of the usual psychometric spatial tests, may not reflect an improvement in spatial ability as such but rather an improvement in the ability to process spatial tasks analytically.

There is another aspect of the psychometric methodology which needs to be noted. Regardless of what spatial factor a test may be attempting to measure, the test item stimuli are almost invariably two-dimensional

figures; similarly, the response required is usually the selection of one or more choices from a multiple-choice set of diagrams.

Spatial ability: the developmental view

Piaget's view of the development of children's spatial ideas typically has the child acquiring topological notions first, followed by concepts relating to perspective, and then euclidean concepts being acquired last. At each level Piaget and his colleagues have devised tasks to ascertain whether a child has acquired the appropriate concepts. Research into the spatial aspects of Piaget's theory is not nearly as common as research on conservation or classificatory behaviour. One particular task, though, has received some attention and this relates to the child's ability to coordinate perspectives; attempts have also been made to train this ability. The task associated with the assessment of the child's ability to coordinate perspectives is the 'three mountains' experiment, described in Piaget and Inhelder (1956). Briefly, the experiment involved 100 children between 4 to 11 years of age and each was shown a model landscape of three coloured mountains; three different methods were used to judge the child's ability to visualize what the landscape looked like from different viewpoints. In one case the child was presented with three cardboard model mountains and asked to reconstruct the relationships with respect to a view of the model landscape as it appeared in a given picture. The other two methods involved the use of pictures of the landscape, the child being asked to choose a picture which represented the view of a doll located at some position round the landscape, or the child was asked to select a picture and indicate where the doll must have to be located in order to take the picture. Most subsequent replications have also used pictures, colour photographs or slides. The results of the studies indicate that the ability to coordinate

perspectives and emerge from a state of egocentricity is a relatively late development although, as Hughes and Donaldson (1979) have shown, the results which lead to this view may be a function of the difficulty of this particular task as 4 and 5 year olds have demonstrated non-egocentric behaviour in less complex situations. The sort of mental processes involved in the coordination of perspectives are, as Eliot (1970) has pointed out, similar to those which Michael et al. (1975) used to describe the psychometric Spatial Visualization factor, although subjects have been shown to use several strategies in solving this task (Huttenlocher and Presson, 1979; Flavell et. al., 1978). Thus, whilst developmental psychologists and the factor analysts have tended to work independently, the results of the studies which attempt to accelerate the development of coordination of perspectives are relevant to those working within the psychometric tradition, and both share similar methodological problems.

VANTAGEPOINTS FOR A REVIEW

In view of the difficulties associated with both the definition and measurement of the spatial ability construct, some care needs to be exercised in the choice of the theoretical vantagepoint from which a wide variety of spatial training studies can be viewed. Thus, to classify studies according, say, to whether the ability trained was Spatial Orientation or Spatial Visualization would present a number of problems, among them the lack of adequate definition of the terms. Several suitable classification modes are available which are independent of a particular school of psychology. An example is the distinction introduced by Guay and McDaniel (1977) between 'low-level' spatial abilities 'requiring the visualization of two-dimensional configurations, but no mental transformation of these visual images' and 'high-level' spatial abilities 'requiring the visualization of three-dimensional configurations, and the mental manipulation of these visual images' (p.211).

Further possible distinctions are Eliot and Hauptman's (1981) dichotomy between spatial ability as an intrinsic aspect of thinking, as in the processing of non-linguistic information, and spatial ability as sets of operations for solving particular kinds of problems; also there is Myer's (1958b) distinction between the 'perceptual' aspect of spatial ability, the ability to 'read graphic language', and the 'logical' or the ability to carry out inferences with these percepts. Clements (1978) has described a system of classifying spatial tasks developed by Wattanawaha (1977), each task being uniquely characterized by assigning it values associated with four characteristics; this system would appear to be particularly useful for investigating the nature of successful and non-successful spatial training programs.

The vantagepoint chosen for this review, however, is drawn from two recent papers by Bishop (1980b, 1980c) who addressed the issue as to what type of spatial abilities might be considered valuable in the context of mathematics. He indicated that 'the need seems to exist for the description and elaboration of abilities which lie within the intersection of the two separate sets of abilities; almost a notion of common abilities' (1980b, p. 14). Bishop then proposed two ability constructs; first, the ability for Interpreting Figural Information (IFI) and, second, the ability for Visual Processing (VP). The IFI ability is clearly one of content and context and Bishop relates it to both the Spatial Orientation factor and Guilford's Cognition of Figural Systems, one of 30 abilities in the figural plane of the Structure of Intellect model (Guilford, 1967, p. 92). The term 'figural information' refers both to the type of item which appears in psychometric spatial tests as well as to the diagrams and pictorial representations common to school mathematics. It is also clear that IFI involves perceptual aspects as well as the knowledge of spatial conventions, the ability to make use of depth cues and make sense of pictures and photographs, all things which

are potentially trainable or which are modified by experience. The IFI ability construct is of use to us here because both the psychometric and developmental views of spatial ability assume this ability and take it for granted when a subject is doing a psychometric test or viewing photographs in a perspective task. The Visual Processing ability appears to be at the heart of spatial ability and it is usually assumed that VP is the means by which subjects solve spatial problems, although the studies of spatial problem solving strategies do indicate that this is not necessarily the case. Bishop indicates that the VP ability involves the processing of not just figural information but non-figural information as well as, for example, in the case of a 'geometric' problem solver using visual imagery to solve a mathematical word problem. Spatial training programs which set out to improve 'visualization' are, by implication, attempting to improve subjects' use of visual imagery in the processing of information or, in other words, the VP ability.

This review, then, discusses the spatial training literature using the perspective of the two spatial-mathematical constructs introduced by Bishop. Those studies considered to belong to the IFI category will be discussed first; the developmental studies, considered to involve both IFI and VP aspects, will be discussed next; and finally, those studies which clearly attempted to train the VP ability will be discussed.

THE INTERPRETATION OF FIGURAL INFORMATION (IFI) ABILITY STUDIES

Studies which can be considered as involving training of the IFI ability or, in Myer's phrase, the understanding of 'graphic language', may be divided into four groups. The first group comprises studies of the perception of pictorial materials, including pictorial depth perception, most of which appear in the cross-cultural literature. The second group comprises studies involving training in

graphics through drawing or drafting exercises. The third group comprises the Soviet studies concerned with training exercises involving the use of diagrams. The last group comprises studies concerned with the effect of practice on various two-dimensional spatial tasks.

Perception of pictorial materials

Miller (1973), in a review of cross-cultural studies of the perception of pictorial materials, investigated the evidence for the role of experience with pictures in facilitating perception and concluded that 'the ability to perceive anything in a pictorial representation requires some experience with pictures in order to acquire the set that pictures can represent more than a flat surface' (p.148). The knowledge of spatial conventions, in particular the depth cues used in pictures, has been related to school experience. Kilbride, Robbins, and Freeman (1968) and Mundy-Castle (1966) found that perception of depth cues was related to the subject's year in school. Clements and Lean (1981), following Deregowski (1968) and Bishop (1978b), studied the perception of depth in both line drawings and photographs of Papua New Guinea urban and rural children and European urban children attending primary schools. For all groups, performance on each task improved with grade in school; also, the performance on photographs was superior to that on line drawings. Both urban groups performed better at all grade levels than the rural children who had little experience of pictorial material and had not the urban children's experience of movies, photographs, television, and other pictorial stimuli.

There are several cross-cultural studies which report attempts to teach pictorial depth perception to children (Dawson, 1967; Forge, 1970; Duncan et al., 1973). Deregowski (1974) found that, for primary school children, even a relatively short exposure to the use of a stereoscope improved the children's depth perception in line drawings.

Leach (1975) found that giving 9 year-olds 15 minutes instruction with a simple apparatus significantly improved their depth perception in photographs, line-and-tone, and line, drawings. Two studies by Nicholson, Sneddon, and Worsnop (1977, 1978) found similar results for high school children, one group responding successfully to stereoscopic diagrams, another to planoscopic diagrams in conjunction with models. Serpell and Deregowski (1972) found that instruction using ordinary pictorial materials found in class textbooks was sufficient to improve primary school children's pictorial depth perception and that this was superior to instruction involving the use of educational films. It is apparent that this particular aspect of the IFI ability, when it does not develop as a result of experience with pictorial materials, may be developed by instruction over short periods of time.

Training in graphics

Each of the studies discussed in this section investigated the effects of a technical drawing or drafting course on subjects' spatial ability as measured by one or more standard psychometric tests. There are two studies which investigated the changes in the scores on the *Space Relations Test* (Educational Testing Service, 1950) with groups of college-level students doing courses in engineering drawing. Worsencroft (1954) found that after an eight month period the mean score of the drawing course group significantly increased while that of a control group did not. Similarly, Blade and Watson (1955) found a significant gain in the mean score of a drawing course group after their first year of study and this was retained at the end of their four year course. Both studies concluded that the change in the mean of the test scores was due to the exercises involved in the drawing courses. Evidence which contrasts with these results is reported by Faubion et al. (1942) who found no significant change in test scores on a surface

development test after a six-week course in mechanical drawing. However, Churchill et al. (1944) reported a significant increase in the mean score on the same test after a nine-week (400 hour) course, and concluded that the most probable explanation of the contradictory results lay in the difference between the amount and intensity of the training which each group received. Churchill et al. also noted the close association between the content of the training program and the nature of the spatial test items. It seems likely that the relatively specific skills developed during the drawing courses resulted in an improvement in the IFI ability and that it was this, rather than a change in the VP ability, which resulted in improved mean test scores.

Two studies of subjects doing architectural drawing or design courses tend to reinforce this view. Stringer (1975) compared the results of two groups on five spatial tests; one group did a course in architectual drawing and the other did not. The drawing group, after six weeks, showed a significant increase in their mean score on the *Differential Aptitude Test-Space Relations*, a surface development test, whereas the other group did not. Stringer concluded that, since the *Space Relations Test* items showed the greatest face similarity to the training tasks, it was the specificity of the training which resulted in the improved mean scores on that single test and not on the others. Lack of transfer to the other spatial tests implies no improvement in the VP ability. McCloskey (1979) reported a study involving two intact classroom groups of high school children, one group receiving design lessons over a period of seventeen weeks during which the control group proceeded with conventional art classes. In this case, both groups showed increases on the *NFER Spatial Tests 1 and 3* although the treatment and control groups mean gain scores were not significantly different. It seems likely that the experience gained in graphics by both groups accounted for the improved performance on the spatial tests.

The overall effect of courses in drawing and exercises in graphics, provided they extend for a sufficient period of time, is such that there is an improvement in subjects' ability to deal with those sorts of figural tasks which are closely related to the training tasks. There is, thus, evidence of near-transfer effects due, most likely, to an improvement in the IFI ability but no evidence of far-transfer which would be indicative of changes in the VP ability.

The Soviet studies

Another aspect of the training of IFI ability appears in volume 5 of the Soviet Studies series published by the S.M.S.G. at Stanford University. In particular, the study by Vladimirskii (1971) deals with two series of experiments which were designed to develop the 'spatial imagination' and the formation of geometric concepts. It was found that a necessary prerequisite for the use of diagrams in the teaching of solid geometry with ninth grade children was that students should be familiar with the rules and conventions of spatial representation. The preliminary work with all the students thus concentrated on this particular aspect. Subsequently, in the exercises which followed, the tasks involved the recognition, composing and decomposing of figures, a variety of questions designed to induce visual processing, and a logical analysis of the graphic material; in other words the tasks attempted to develop both the IFI and VP abilities as well as an analytical approach to problem-solving. Vladimirskii claims a degree of success through the use of his exercises; the numbers of subjects used, however, we infer to have been relatively small. It would appear that there was an improvement in the subjects' IFI abilities; it is less clear, however, how far the subjects' VP abilities were developed. Most subjects experienced difficulty while doing the exercises and Vladimirskii observes that this together with 'the short period during which the subjects worked on them lead us

to conclude that to develop spatial imagination one needs a system of exercises which extend over a long period' (p.93).

The effects of practice

Within the psychometric tradition, a common method of attempting to improve spatial ability is that of straightforward practice on solving spatial tasks similar to, or precisely the same as, those found in spatial tests. The emphasis is on acquiring skill in dealing with a particular sort of graphic information, thus devleoping the IFI ability, as well as providing the subject with an opportunity for developing a strategy, perhaps analytic, for solving a limited set of spatial tasks. Connor and her colleagues have carried out several such studies which have involved attempts at improving subjects' performances on the *Children's Embedded Figures Test*. The first study (Connor et al., 1977) involved primary school children in grades 1, 3, and 5 in two treatment groups and one control group. The treatments comprised different approaches to disembedding figures from extraneous detail, carried out over short periods of time. The post-test results indicated that only the females in one treatment group had benefited from the disembedding practice. A second study (Connor et al., 1978b) however, involving first grade children, reported that both boys'and girls'post-test scores indicated the beneficial effects of practice. A third study (Connor et al., 1978a) with educable mentally retarded children found a pre-test to post-test gain that corresponded to approximately four years of maturation in non-retarded children.

Further studies by Goldstein and Chance (1965), Krumboltz and Chrystal (1960), McGee (1978), and Drauden (1980) all report significant practice effects on a variety of standard spatial tests. The evidence thus indicates that practice at dealing with a narrow range of spatial tasks can improve a subject's performance on those tasks presumably through the development of simple solution

strategies whether visual or analytic. Drauden's (1980) study indicates, however, that there is little or no transfer value in such practice.

The studies discussed in each of these four sections have dealt with a variety of spatial skills which can be considered aspects of Bishop's IFI ability. The evidence from each indicates that these various skills are trainable given the appropriate experiences. Brief training with pictorial materials is sufficient to induce pictorial depth perception; relatively brief practice is sufficient to improve subjects' performances on spatial test items; the teaching of various spatial conventions and exercises with diagrams helps to improve geometry performance; and sufficiently long experience with technical drawing appears to be sufficient to improve performance on spatial tasks similar to those done in the drawing course.

STUDIES FROM DEVELOPMENTAL PSYCHOLOGY

The training programs associated with the attempts to accelerate children within the Piagetian projective developmental stage and focusing upon variants of the three-mountains task usually assume that it is the VP ability that the program is trying to change. It is often overlooked that aspects of the IFI ability are called into play in that the use of pictures, drawings, or photographs is part of the experimental method. By learning the appropriate rules of how pictures of an array of objects would alter as the viewing position altered, a subject could avoid using a 'mental rotation' strategy; a training program may thus succeed due to an improvement in the IFI ability rather than to any change in visual processing. Piaget's criteria for judging whether or not a training program has succeeded (Piaget, 1964) include the conditions of retention of the trained skill over a period of time, and of transfer to other operations, a condition which, if fulfilled, would presumably indicate that a change in VP ability had occurred rather than just a change in IFI ability.

Of the various attempts to train the coordination of perspectives, most studies can be regarded as unsatisfactory either because they report no significant improvement or because they fail to satisfy one or both of the Piagetian criteria. Eliot (1966), for example, concluded that there had been no significant change in the performance of young children, after six hours training, to predict the camera positions of photographs taken around a model mountain array. Miller, Boismier, and Hooks (1969) reported limited success with second grade children after six sessions of activities but neither retention nor transfer tests are reported. Arond (1966) reported some success in training a relatively simple left–right reversal task, with eight–year–old children, and with a retention test administered two or three days afterwards; some near–transfer effects were also reported. Miller and Miller (1970) provided a retention test seven months after training but did not test for transfer. A further study by Miller (1977) reports significantly improved performances on a perspective task by kindergarten children after eight training sessions of an hour each. Miller, however, does not report retention or far–transfer tests.

A study which does adequately satisfy the two criteria is that of Cox (1977, 1978) who has carried out the longest and most detailed of the training programs. Working with children almost six years old, the program for each child comprised twenty sessions, each fifteen minutes long, spread over forty days. Cox reports that the trained group had significantly higher mean scores than the control group on the criterion perspective tasks and that this difference was maintained at the first retention test seven weeks later and at the second retention test seven months later. There was also evidence of limited transfer, not significant, to other tasks.

The training programs which have been successful, even if not fully satisfying the Piagetian criteria, were concerned with providing young children with a carefully sequenced

set of activities over an extended period of time usually in a clinical, interview situation. That an aspect of the IFI ability of the subjects was improved is likely due to the considerable emphasis placed on the use of pictures in doing the perspective tasks. It is more difficult to judge, however, to what extent the VP ability was changed except perhaps in Cox's study where the evidence of some transfer occurring may be indicative of a change in processing skills.

THE VISUAL PROCESSING ABILITY STUDIES

Studies will be discussed here which have set out to improve 'spatial ability' in general, or 'spatial visualization' in particular; the aim implicit in these studies is to improve a subject's use of visual imagery or visual processing skills, in other words Bishop's VP ability, in the solution of spatial tasks. The criterion tests for judging the success of these programs are the various standard psychometric spatial tests; often there is some inconsistency between the stated aims of the program and the reference tests used to judge its effectiveness. We have, for example, Van Voorhis' (1941) study which sets out to 'improve the power of visualization' (p.62) and which uses reference tests which measure the Space Relations (S–R) factor. Other methodological problems associated with the various strategies used to solve spatial test items and with the investigation of visual imagery and processing have been mentioned above; their relevance here is that it is difficult to know whether changes in spatial test scores, which occur after some training program, are due to changes in IFI ability or to the development of analytic solution strategies or to a change in the VP ability. By invoking the Piagetian criteria of retention of skills together with transfer to other tasks, we obtain a means of inferring changes in the VP ability. With these criteria in mind, two groups of studies are discussed; the first group comprises

studies which have investigated the effect of various mathematics courses on spatial ability; the second group comprises studies in which training programs have been specifically devised to attempt to improve spatial ability.

Mathematics courses and spatial ability

Ranucci (1952) found that a five-month course in solid geometry produced no significant changes in the performance of high school students on four tests of the S-O and Vz spatial factors. Brown (1954) concluded that high school courses in plane and solid geometry did not significantly effect performance on the *Differential Aptitude Test - Space Relations*. Similarly, Cohen (1959) found that a five month high school course in solid geometry, which involved the construction of models, produced no significant differences between the experimental and control groups in the performance on three standard tests of the Vz spatial factor. The only study in this section which reports some success is that of Bishop (1973) who found that a sample of boys who had used structural apparatus as part of the Nuffield primary mathematics course performed, in seventh grade, significantly better on the *NFER Spatial Test 1* than those who had not. Bishop also reports a subsequent experiment in which 23 high school girls showed increased scores on two spatial tests after four training sessions involving manipulative apparatus (Bishop, 1973).

Bishop's study involved a spatial training program given in a mathematics context rather than a mathematics course as such which the other studies did. The various geometry courses were not successful in bringing about changes in spatial test scores whereas Bishop's relatively brief period of activities with apparatus did appear to result in some success. These results may also indicate an age factor; the three geometry courses were all given to high school seniors whereas Bishop's study involved primary and seventh grade children.

Burnett and Lane (1980) report a study carried out with 142 male and female college students who were given the Guilford – Zimmerman Spatial Visualization Test before and after two years of college study. Those students majoring in the humanities and the social sciences improved less than those majoring in mathematics and the physical sciences. A significant correlation was found between the number of mathematics courses taken and the difference between the pre– and post–test scores. The authors contend that this result supports the notion of the modifiability of spatial visualization skill and, further, that the content of mathematics courses is an important contributing factor in the improvement of spatial test scores. The authors, however, do not speculate as to what it might be in the content of the courses which appears to affect the spatial test scores. One possible explanation of the results is that the subjects, during the two year period of their college course, increased the scope of their analytic problem–solving skills and were able to apply these to the solution of the spatial test items. If this were the case, then the improvement in the spatial test scores does not reflect an improvement in spatial visualization skill. In the absence of information regarding the nature of the subjects' strategies used in the solution of the test items, the results of this study cannot be taken as conclusive evidence in support of the modifiability of the VP ability.

Spatial training programs

Wolfe (1970) devised a 'Space Visualization Training Program' which was given to students in grades 7, 8, and 9 who were also pre– and post–tested on tests of the S–O and Vz spatial factors and on an arithmetic test. The results indicated that there were no significant differences between the experimental and control groups at all grade levels on the test of the Vz factor. The grade 8 experimental students, but not the others, scored

significantly higher than the control or the test of the S—O factor. There were no significant differences, at all grade levels, between groups on the arithmetic test. In Wolfe's program, only one session out of ten was specifically devoted to the sort of visual processing skills which may be used in the solution of Vz tasks; it therefore seems likely that the limited success of the grade eight group was due to an improvement in an aspect of the IFI ability, or what Guay and McDaniel (1977) call 'low—level' spatial abilities.

Similar comments appear to apply to studies by Van Voorhis (1941) and Brinkman (1966). The first study involved college freshmen in a program comprising 12 one—hour lessons spread over 12 weeks. Van Voorhis concluded that the experimental group had benefited from the training program on the basis of their performance on tests of the S—R spatial factor and a descriptive geometry test. An analysis of the program reveals that a large proportion of it was devoted specifically to exercises involving the use of figural stimuli; this together with the results that improvements occurred on S—R spatial tests indicate that a change in the IFI ability has occured. Brinkmann's study reports the results of grade 8 children who worked through a self—instructional, 550—item linear program specially designed to 'teach the visualization of space relations' (p.180), and which was spread over approximately three weeks. The content of the program, as described by Brinkmann, appears to be aimed specifically at improving subjects' handling of graphic stimuli, assisting the development of strategies appropriate to dealing with 'low—level' spatial tasks. The results, which were such that the experimental group scored significantly higher than the control group on the *DAT-Space Relations* test, are probably indicative, therefore, of an improvement in an aspect of the IFI ability.

Mundy (1980) devised a spatial training program for college students which aimed at improving their spatial abilities and as a result it was hoped that this would bring about improved performance on tests of calculus topics. Mundy's experiment involved 250 subjects (123 female and 127 male) assigned to seven groups under three treatments: Audio-Visual (AV), Audio-Visual-Tactual (AVT), and Control. The AV training program consisted of a self-paced, programmed tape-slide presentation in six modules dealing with aspects of two- and three-dimensional spatial tasks of the sort found on a variety of psychometric spatial tests. An attempt was made to provide subjects with a number of solution strategies of the type reported in Barratt (1953). The AVT program was identical to the AV program with the addition of associated concrete materials. The subjects were pre-tested on their calculus background and the criterion test of spatial ability used was the DAT-Space Relations. The post-tests comprised two calculus tests and the DAT-SR; these were administered eight weeks after the pre-test and during this time all subjects took a course in calculus and the subjects in the training groups completed their six training modules. The results indicated that, for the DAT-SR, a significant practice effect was apparent but that the training effects were not significant. A significant sex x treatment interaction was found for the calculus tests which indicated that the AV training was more beneficial for females than for males.

Two studies which involved a variety of concrete activities are those by Saunderson (1973) and Wearne (1979). Saunderson, working in Papua New Guinea, carried out a five-hour training program with 40 first year teachers' college students. The program, spread over three weeks, involved no formal instruction and students worked individually or in pairs on a variety of games and activities. The results were presented in terms of a weighted mean with the major weighting assigned to the *Minnesota Form Board* test; the experimental group

obtained a significantly higher mean than the control group. The data relating to the individual means of the four post-tests are not given but it would not be surprising if the greatest increases had occurred on the *Form Board* test since the majority of activities in the program involved the construction of plane shapes, which is essentially what the test items require. Also, since Guay et al. (1978) have shown that 'analytic' strategies are almost invariably used to solve *Form Board* test items, it is possible that improvements in scores on this test were due to improved analytic solution strategies rather than improved visual processing skills. The study by Wearne (1979), which involved a six-hour training program with a wide variety of activities, reports an attempt to determine the solution strategies used by his grade 8 subjects in solving items on the *Monash Space Visualization Test* which was the pre- and post-test instrument. At the end of the training program, Wearne reports that the subjects who had made the greatest gains in test scores had demonstrated an increased number of analytic solution strategies.

Clements (1981) reports a study by Rowe who carried out a large spatial training study involving grade 7 students with two treatment groups with 63 subjects each and a control group with 79 subjects. One treatment group received a two-dimensional training course involving, in Guay and McDaniel's term, 'low-level' spatial skills. The other treatment group received a three-dimensional training course involving 'high-level' spatial skills. Post-tests were administered immediately after the completion of the training and also one year later. In this respect Rowe's study is notable in that it is the only study discussed here which reports data on retention of trained spatial skills. The results indicated that both treatment groups performed equally well on the post-tests and significantly better than the control group. An analysis of the results indicated, however, that the

the improvement of both treatment groups was on tasks involving 'low-level' spatial skills and that neither group showed significant improvement on tasks requiring 'high-level' spatial skills, i.e. three-dimensional thinking and the mental manipulation of images.

Finally, there are several recent studies which have investigated the relationship between subjects' spatial abilities and their behaviour in solving mathematical problems (Handler, 1976; Schonberger, 1976). A study which has addressed this question and, in addition, has investigated the effects of a spatial training program on problem-solving performance is that of Moses (1979). The thirteen-week training program taken by two fifth grade experimental classes comprised thirteen one-hour sessions consisting of activities involving both two- and three-dimensional tasks. The treatment classes and the two control classes were pre- and post-tested on measures of spatial ability, reasoning, and mathematical problem-solving performance, and Moses' analysis of the post-test scores indicated that the training program had apparently significantly affected the scores on all tests. The effect of training on problem-solving performance was such that improvement was greater for problems which were more spatial in nature than on problems which were more analytic; there were no significant differences in training effects on problem-solving performance between students of high and low spatial ability. These results are similar to those of Moses' earlier doctoral study (1977) in which an eleven-session training program was used and this was found to signficantly affect spatial test scores but not, in this case, problem-solving performance. Whilst Moses' studies do not report retention data we do have data reported on the transfer of training in that the effects of training on a reasoning test and problem-solving performance are given. In the absence of clinical investigations such apparent transfer effects are the only means available for inferring changes in the

VP ability. Moses' work is important from the point of view of the mathematics educator in that it provides some evidence of the apparent usefulness of spatial training programs on children's performance in solving non-routine mathematical problems.

The majority of the studies discussed in this section have not been concerned with either retention or transfer and thus it is not possible to judge whether changes in subjects' VP abilities have occurred. A number of studies have clearly been successful in improving subjects' scores on psychometric spatial tests. Unlike the studies involving drawing training or practice, which provide experience in a narrow range of skills resulting in improvements in solving a narrow range of spatial tasks, studies like Rowe's have provided a wide range of spatial activities resulting in improvements in scores on a number of spatial tests. School geometry courses have not been successful as spatial training programs, presumably because they did not include a sufficient number or variety of spatial activities. Except for Moses' study, in those cases where we have evidence of significant improvements on standard spatial tests, this would appear to be due either to an improvement in 'low-level' spatial skills and, thus, in the IFI ability, as in Rowe's study, or an increase in the use of analytic strategies in solving spatial tasks, as in Wearne's study.

CONCLUSIONS

This review has adopted deliberately a wide view of those abilities which may be termed spatial and which appear to have some relevance to mathematics education. These abilities encompass, on the one hand, the skills necessary to the reading, interpretation, and understanding of the sort of pictures, drawings, and diagrams which appear in mathematics textbooks and classrooms, as well as a range of 'low-level' spatial abilities necessary to the solution of some transformation geometry problems as well as S-O

space factor test items. On the other hand, we have a range of 'high-level' spatial abilities involving the mental manipulation of visual images and, in general, the ability to process a wide range of information in a visual way. In discussing studies which fall into these two classes of abilities we have adopted Bishop's distinction between the Interpretation of Figural Information ability construct, which encompasses the former class, and the Visual Processing ability construct which encompasses the latter. There appears to be abundant evidence indicating that various aspects of the IFI ability can be trained but little conclusive evidence that the VP ability can.

The evidence from cross-cultural studies indicates that, for those children who have not been regularly exposed to pictorial stimuli, suitable training programs, usually of brief duration, may be devised for remedying pictorial object recognition and pictorial depth perception. Vladimirskii (1971) has also shown that suitable exercises are able to improve children's understanding of diagrams and spatial conventions associated with geometry.

Much evidence is available which indicates that it is possible to train 'low-level' spatial skills, particularly the type associated with many standard psychometric spatial tests of the S-O factor. Not surprisingly, practice at solving a particular spatial task generally results in improvement on the performance of that type of task. Technical drawing courses may also bring about an improvement in performance on those tasks which involve skills similar to those acquired in the course. School geometry courses were not found to be successful in improving this type of skill. However, training programs which involved a wide range of spatial activities did result in improved scores on a number of spatial tests which indicated, as Rowe has found, an improvement in 'low-level' spatial skills. We also have evidence that improvement in spatial

tests scores may be due to improvement in subjects' analytic strategies in the solution of spatial problems.

It is difficult to find clear, unambiguous evidence that it is possible to train the VP ability. Most of the Piagetian studies which attempt to do this fail, however, to satisfy the two conditions of retention and transfer. Studies from within the psychometric tradition similarly fail to meet these two criteria; it is possible that a number of these studies were successful in bringing about changes in the VP ability but without data gained from retention and transfer tests it is not possible to reliably infer that it has. Moses' (1979) study does provide evidence that a spatial training program with grade 5 children had apparent transfer effects on reasoning and problem-solving performance.

The children in the successful studies by Cox, Moses, and Rowe were respectively, five-year olds, grade five, and grade seven students. Studies with older subjects have usually had limited or no success. Thus it may well be that spatial training programs stand a much greater chance of success with younger children and would best be carried out at primary school level.

From the point of view of mathematics education it appears likely that an increased emphasis on visual methods in mathematics, i.e. the teaching of spatial conventions and the reading and interpretation of pictorial representations as well as the use of diagrams in problem-solving, at primary level, in particular, and the high school as well, would be beneficial for some students and would counteract the bias towards largely analytic approaches to mathematics teaching. In addition, general programs involving a wide range of spatial activities, incorporated into the primary or lower secondary curriculum, appears likely to improve children's performance on a range of spatial tasks, particularly of the 'low-level' variety.

Future spatial training programs which aim at changing the VP ability need to attend to a number of aspects both with respect to experimental design and to the nature of the training program itself. In most of the studies reviewed here, pencil and paper psychometric tests of spatial ability have been used as the criterion instruments for judging whether changes in the VP ability have occurred as a result of training. As has been pointed out previously, an improvement in spatial test performance may occur due to the acquisition, or more efficient use, of analytic solution strategies. In order to overcome this difficulty it may be necessary to use those spatial tests which are known to elicit only spatial solution strategies or invoke a subject's VP ability. Alternatively, the spatial tests should be administered in conjunction with a solution strategies questionnaire of the type devised by Coulson and Burden (1982). A further possibility is not to use the psychometric tests at all but rather determine a subject's use of the VP ability in an individual, clinical interview, using perhaps items selected from the tests. The clinical approach may also be useful in attempting to determine what aspects of the training program itself have been beneficial, a point which has received no attention whatsoever in the literature. Finally, in the experimental design, the testing program should be devised such that it is possible to investigate whether near and far transfer effects are apparent, and the extent to which retention occurs.

Certain aspects of the training program itself need to be addressed. The training programs, reviewed here, which have attempted to change the VP ability, all appear to lack one potentially important feature, i.e. the dynamic display of various transformations of spatial representations. One essential aspect of the VP ability involves the mental, dynamic transformation of visual images. It thus may be useful, in future training programs,

to make use of films, film-loops, and the micro-computer with its associated graphics soft-ware. These could perhaps be introduced after subjects have had experience with concrete materials and they may be beneficial in assisting the internalization process in the development of dynamic visual imagery.

From the point of view of mathematics education, those training programs which aim at improving mathematics performance may well be more effective if they attempt to train a range of relatively specific visual skills which may be utilized in problem-solving rather than attempting to train the VP ability itself in the hope of obtaining some beneficial spin-off in terms of improved mathematical performance.

It has been pointed out that the relationship between spatial ability, visual imagery, and mathematics performance is not clear and, in particular, that there are theoretical and methodological problems associated with the spatial ability and visual imagery constructs. Further, it has not been possible to find clear, unambiguous evidence that spatial training programs have produced changes in the VP ability, and yet the interest in such programs usually assumes that some sort of change in visual processing skills results from training and that these skills are used beneficially in reasoning and problem-solving. Until there is further evidence from studies like that of Moses as well as detailed clinical investigations, such a view is unsupported.

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DRAWING THE WORLD AND NAMING ITS PARTS, AND DRAWING THE NET OF A SOLID: A CASE STUDY OF SPATIAL ABILITY TASKS FOR MATHEMATICS TEACHERS AND STUDENT TEACHERS

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ABSTRACT: Twenty five mathematics teachers and student teachers were given the Piagetian task of drawing the two-dimensional net for a three dimensional solid object. The shapes ranged from such simple objects as a cube to such complex objects as a cylinder and a cone.
The errors that were made indicated probable limitations in the subjects' own educations and in their ability to teach high school mathematics.
Subjects were also asked to draw a map of the world from memory and to name as many countries and capital cities as possible. The map-drawing was used to measure subjects's large scale spatial ability, and the naming was used to measure a related analytic-verbal ability.
Some comparisons were made between the results for the net task and the maps tasks.

Problem Solving

Adapted for Yapese Elementary Schools

Grades 1, 2 and 3

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In 1980 the National Council of Teachers of Mathematics in the United States declared the Eighties to be the Decade of Problem Solving. That same year they published their yearbook entitled Problem Solving in School Mathematics. The set of steps recommended were:

1. Understand the problem - Identify the data and the unknown. Recognize the condition.
2. Devise a plan - Find a connection between the desired answer and the data. It may use theorems, definitions, or a known process used on other problems previously worked.
3. Carry out the plan - Use the rules of algebra, geometry, etc. to solve the plan.
4. Examine the solution - Evaluate the reasonableness of the answer and check the work if it is questionable.

These steps have evolved in large part from the work of George Poyla, the eminent California mathematics educator in the Fifties and Sixties.

In November, 1981, we set out to determine the competency in problem solving in the Yap schools using a series of periodic progress tests that we were administering throughout the island. Each test was a mixture of basic facts, computations, measurements

and word problems, with about one-third of the questions concerned with application. The average for the entire test was 52%. However, when we graded only the questions dealing with applied problems, and treated them as a complete test, the average was only 30%. With that extensive difference between the means, the t value was so exaggerated that there could be no doubt that ability in problem solving lagged far behind development in basic computations. Closing that large gap in our curriculum became our major concern.

When we visited the teachers in their classrooms and asked specific questions related to problem solving, all of them admitted that it was the most difficult topic for them to teach. In fact, most of them were confused about what they were supposed to be doing in the problem solving lessons. This isn't surprising since most of our teachers are untrained in educational methodology, and since they are obliged to use texts printed in the United States which are difficult to adapt to local situations. Even though we use one of the best texts, its presentation of problem solving uses an ill-defined process with an obscure relation to that of the National Council. This paper is the result of our effort to adapt those processes for use in our classrooms in Yap.

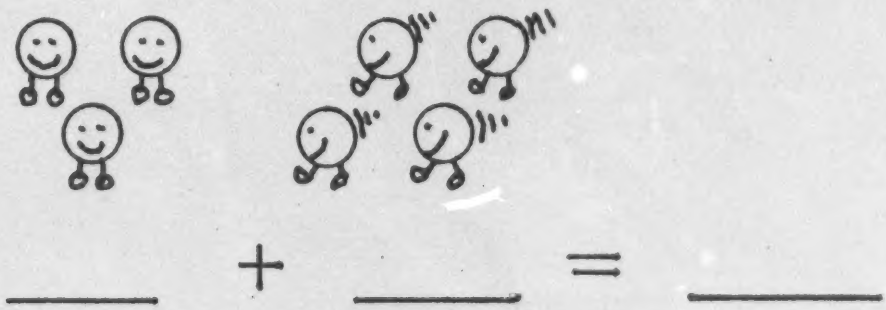
Taking into account both our teachers' lack of sophistication and the fact that our students are learning through a second language, our adaptation tries to be as simple as possible, and at the same time as explicit as possible. Like the National Council, we have four steps in our method. Our first two steps

detail the first step of the National Council's recommendation. Our third step corresponds to their second; and our fourth to their third. We only mention their fourth step as an afterthought. Our arrangement is:

1. Numbers - Identify all of the numbers along with their units, including the question, which is an unknown number with units.
2. Operation - Identify key words that translate as mathematical operations or relations, and write them with the appropriate numbers.
3. Statement - Organize the numbers with their units and the operations into a mathematical statement, usually an equation.
4. Execution - Apply the rules of arithmetic to perform the operations that solve for the unknown number.

The following examples from the first three grades illustrate these steps in use.

I. In the first grade picture problems are presented like:



Although everything seems obvious and most children just count impulsively, we require them to consciously go through the

steps so that they can develop a system for understanding problems. Because the solution to a problem won't always be so obvious, we must teach the students to take the time to go through the process, step by step, performing the operation last. In the first step the children identify the numbers and write them down while naming the "units".

3 (circle people)

4 (circle people)

? (how many circle people)

It is important to make the children aware that story problems are about numbers of things, "circle people" in this case. Also, that even the question will be a number of something, and it can be inferred in this problem to be the same as the other two numbers because their units are identical. This could be considered an axiom of addition and subtraction problems, which leads into the next step.

The second step is to decide what operation to use. Again it is obvious in this problem because the addition sign is already written, but our purpose is to give the children a reason for having that operation. The class should be made conscious that addition is implied because the second group of circle people is going toward the initial group. The idea and words "going toward" and "being added to" are emphasised both in English and in the vernacular to stress their connection to the addition sign. After the fact of addition has been established between the two numbers, the question is presented, "What are we supposed to find?"

The answer, "How many circle people altogether," is also emphasised because the idea "altogether" should be synonymous with "the answer to addition."[1] Then later, when no plus sign is written, but a question with words asks, "How many altogether?" the fact that this unknown number will be the answer to addition clues the operation necessary to work the problem. This step is probably the most important in understanding what the word problem is about. Second language learners seem to pick this up quite naturally once they have been directed to translate words into operations. Possibly this is because they have already developed the habit by translating English and their vernacular.

Now that the numbers are known and the operation has been determined, the third step is to organize them into an equation. For this problem it merely requires that the students fill in the blanks on either side of the addition sign with "3" and "4," the numbers coming together, or being added; and letting the remaining blank on the other side of the equal sign represent the question. Of course, that could have been done at the beginning, and that is all many of our teachers used to do, but then the students would be ignorant of any problem solving process. It is important to keep in mind that problem solving is a whole process involving a series of steps, not just the solution of an equation.

The fourth step is simply the addition fact, 3 + 4 = 7, then, using the unit, the complete answer is 7 . Finally the

[1]"Altogether" only means "the answer to addition" when the numbers have the same units. If the units are different, for example, three rows of four oranges each, "How many altogether?" signifies "the answer to multiplication".

answer is examined to check if it is reasonable.

This same procedure is followed during drills at the beginning of classes when word problems are given orally.

II. In the second grade problems are given like:

20 children playing
3 of them left
How many were left?

Identifying the numbers first, the students write:

20 children
3 children
? children

Notice that "3 of them" has been explicitly written as "3 children," and that "How many" has also been clarified as "? (how many) children." Although language often assumes ideas that have been previously mentioned, in mathematics it is better to name things unequivocally. When we were introducing this method into our schools, most students understood this step quickly for it is really quite easy. For many it was the first positive experience that they had ever realized while working word problems, and this gave them confidence to continue through the process.

To help with the second step, each class develops a Math-Word chart to post in their room. The chart lists important words that the class has found in word problems, and groups them according to which operation or relation they indicate.[2] In this example, "left" indicates "- n (subtract number)" and "were left" implies

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[2]An example fifth grade chart is on page 11.

"- = n (the answer of subtraction)." These operations are written with the numbers like:

20 children
- 3 children
= ? children

This is usually sufficient to enable the students to write a subtraction equation in the next step. But some of the students may be interested in the understood idea that "20 children (altogether)," which can signify "+ = n (the answer of addition)," translates as "n - (number subtract)" in this problem. The class has already learned that the two expressions are equivalent and that the location of the number changes this way for the inverse operations.

By starting the process with the easy task of naming the numbers, then following that with the translation of words into mathematical symbols, which involves a familiar and natural activity for our second language learners, the students work through the first half, and the most critical part, of the process by applying previously acquired skills to new and unfamiliar situations. Not only are the students employing one of the fundamental premises of problem solving, but they are also discovering that problem solving can be fun and easy.

After the first two steps are completed, there is generally little difficulty organizing the mathematical statement:

20 children - 3 children = ? children.

Again, the process of finding the equation is the object of

teaching problem solving, not simply writing and working the equation. Often when students try to solve problems, they begin with the third or fourth step; consequently, they don't understand the exercise completely.

The execution is simply the subtraction, 20 - 3 = 17, and the full answer, 17 children, is examined to determined whether it is reasonable.

III. A sample third grade problem is:

> Rungun works 5 hours every Saturday helping at her family's store. How many hours does she work in four Saturdays?

Initially the children write the numbers:

> 5 hours
>
> 4 Saturdays
>
> ? hours

In order to determine the operation for this problem, we again refer to the Math-Word chart where words such as "every," "each," "an" and "per" indicate rate, a ratio of measures. Normally this concept is not introduced until the fifth grade, although simple fractions like $\frac{1}{2}$ and $\frac{2}{3}$ are discussed as early as the first grade. Because the notion of rate simplifies problems involving multiplication and division, we extend the childrens' understanding of simple fractions to include simple rates such as $\frac{\text{km}}{\text{hr}}$ (kilometres per hour) and $\frac{\text{eggs}}{\text{basket}}$ (eggs in a basket). In this example the expression "hours every Saturday" represents the rate $\frac{\text{hours}}{\text{Saturday}}$. Writing this with the numbers gives:

$\frac{5 \text{ hours}}{\text{Saturday}}$

4 Saturdays

? hours

Although we have completed the second step by translating the key word and writing it with the appropriate number, the operation of the problem still isn't obvious. Rather, it's not in a form that we are used to seeing because there is no operating symbol. But the units along with the rate give sufficient information to construct the mathematical statement, and this needs to be taught so that students will be able to recognize it as multiplication. The following theorem explains this organization:

> The number with the rate is multiplied with the number whose unit is the same as the denominator of the rate, and the product is the number whose unit is the same as the numerator of the rate.[3]

This may seem like a complicated rule for a third grader, but the children learn it more easily than they learn renaming, carrying in addition and borrowing for subtraction. The organization for this problem is:

$$\frac{5 \text{ hours}}{\text{Saturday}} \times 4 \text{ Saturdays} = ? \text{ hours}.$$

The final step, executing the operation, requires both the multiplication fact, $5 \times 4 = 20$, for the numerical part of the answer, and the cancelling theorem of fraction multiplication to justify the answer's unit that is already written. But that is

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[3]This theorem can be proved in the fifth grade. Before that students just agree with their teacher's argument.

just a corollary that the class learns when the teacher explains the mechanics of the previous theorem. Finally the students should consider whether the answer, 20 hours, is credible. A good way is to reread the problem substituting "20 hours" for "How many hours."

Since we introduced this method, the difference between the general test average and the average of just the applied problems has been reduced almost one-half. The breakdown of steps has given both our teachers and our students a new awareness of word problems. The translation of words into mathematical symbols is the focus of this process. A Math-Word chart showing correspondence appropriate for each grade, although unable to give a complete catalogue of translations, does give some of the more frequent relationships thus allowing the students to expand on the concept. So far we have only applied this method to basic arithmetic and algebraic situations. It will probably need to be modified for nonroutine problems.

MATH-WORD DICTIONARY

ENGLISH	+ - × ÷ =	THIN NU WAAB[4]
number AND PLUS MORE JOINS	n +	namba NGE PLUS NAG KU BOCH KU BA YAY
number IN ALL ALTOGETHER TOTAL (all units are the same)	n = + n -	namba GUBIN URNGIN KE MAN (taab rogon e measure)
number LESS MINUS	n -	
number LEFT GONE USED verb AWAY	- n	namba KE YAN KE MUL KE LUF KE THAY BOCHUUW
number verb DIFFERENCE REMAINDER LEFT	n = -	namba KE MAGAY KE PAR KE AW KA BAY
fraction OF number TIMES AT	n ×	namba TIMES NAG
number IN ALL ALTOGETHER TOTAL (the units are different)	n = × n ÷	namba GUBIN URNGIN KE MAN (ba thil e measure)
number SEPARTED INTO DIVIDED BY	n ÷	namba FO'TH DIVIDE NAG
number DIVIDES	÷ n	
number PER measure A AN EACH FOR EVERY	$\frac{n}{m}$	namba U BA measure U REB KO
AVERAGE	total ÷ number	
IS ARE WAS WERE EQUALS	=	E TAAB ROGON KO

[4] Yapese language

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RESEARCH REPORT 1982- 1983.

Project title: MONITORING STANDARDS IN MATHEMATICS OF THE GRADE 10 OUTPUT

L. ROS - GOROKA TEACHERS' COLLEGE

Objectives:

The project tries to identify patterns relating to:

a. Actual item achievement
b. Comparison of students entering various institutions
c. The validity of criteria used for grade 11 selection
d. Comparison of the quality of the intake with previous intakes.

Methodology:

Two tests, D_2 (Arithmetical Ability) and A_4 (Arithmetical Skills) are administered to large groups of Grade 11 students or the equivalent. A D_1 test was developed when I was teaching at Keravat in 1973-74. It was designed to give a mean of 25 and a S.D. of 10 for Papua New Guinea students at the grade 11 level. The D_1 test was slightly modified in 1976 and called D_2 because algebra as such had been excluded from the Provincial High School Syllabus.

The test includes: numberfacts, place value, order of magnitude, patterns, ratio and proportional parts, and tests of confidence in working with numbers. The inclusion of many problems on proportionality and sequences was based on Jones' findings (E.R.U. report 6, 1973), that Preliminary Year results in Maths at UPNG have the best overall correlation with proportionality test items.

In November 1975 Dr. D. Tombs, then Head of Mathematics at the PNG University of Technology (PNGUT) in Lae, suggested a follow up of the D_1 test, which would concentrate on skills only.

The outcome was the A_1 test, designed by Dr. D. Rasmussen, also of the PNGUT Department of Mathematics, and based upon the Scientific Research Associates Computational SKills Development Kit. This test covers the four Arithmetical operations as applied to natural numbers, integers, valgar fractions and decimals. Similar tests, viz A_2, A_3, and A_4 were subsequently designed to serve as pre and post tests in remedial courses.

The tests are administered in January/February in National High Schools and Tertiary Institutions before any remedial work is done. The results are then analysed. The analysis gives the means and standard deviations for each institution and a correlation coefficient, and also the overall mean and standard deviation. In addition an item analysis is done for the D_2 test results. The A_4 results are analysed in a percentage matrix. The figures in the matrix show the percentage of students who master the particular skills. Mastery is defined as: scoring completely correct all problems in a particular section of the test.

Validity and Reliability:

The results of the D_1 test were correlated in 1975 with (i) the results of the Pacific Numberical Reasoning Test, (ii) Grade 10 school leavers results in Mathematics, (iii) first semester mathematics results for students at the University of Papua New Guinea at the Waigani and Goroka campusses. The correlations were all very high, between 0.60 and 0.81. In addition the results of the D_1 test correlated highly (0.65 and 0.51 resp.) with both performance in mathematics at UPNG and with the Psychology Department Test (JPM). Dr. Leonie Ryder, formerly Chief Psychologist with the Public Service Branch (Public Service Board) called the D test "the best predictor of success in mathematics studies". The results of the D_2 test were correlated in 1980 with progress of students at Goroka Teachers' College in the Arithmetical Skills course during the first semester. The correlations with results of 4 consecutive tests, the last of which was on measurement, were 0.76, 0.78, 0.73 and 0.79.

Findings:

The results are published each year and sent to schools and institutions. The project has received the support and recommendation of the National Institutions Division and of the Research Branch of the Department of Education.

The data appear in Tables IX, X and XI.

Comments: 1982

1. The results of both the D_2 test (Numerical Ability) and A_4 (Numerical Skills) have remained fairly constant at approximately 24 and 33 over the years. (Table IX).
   The overall percentage of students who master the particular skills as calculated from the percentage matrix, remained constant at 48-49. The figure 43% for 1982 is not reliable since for the Waigani sample school leavers and non school leavers were included. The results of the non school leavers must have brought the figure down. (Table XI).

2. The meaning of distinction, credit, upper pass and pass in terms of D_2 scores remained also constant at respectively 31, (S.D.6-7), 24 (S.D. 7-8), 18 (S.D. 6-7) and 13 (S.D. 4) (Table X).

3. In the 1981 analysis the results of Passam and Aiyura had been combined, since the results from Keravat were sent at a later date. When they came in they showed again a striking similarity to the combined results of Passam and Aiyura. It was known however that the Aiyura intake has been of a poorer quality than that of the other National High Schools. (Table X).
   The 1982 analysis showed that although the overall mean for the D_2 test has remained constant at 24.23 the quality of the intake for the various institutions differs substantially, in particular the quality of students admitted to Waigani and Goroka (School leavers only). (Table IX). Some National High Schools have a better quality intake than others in terms of D_2 scores, but also their distictions are of a better quality than those in other schools.

4. Various trends appear in the item-analysis and the percentage Matrix. It would be premature to draw conclusions at this stage.

RESEARCH REPORT 1983

For 1983 we regret to say that two National High Schools did not co-operate despite the request made by the National Institutions Division. (File EC2-4-13, letter dated 27th October, 1982). In one of the two schools the test papers were (as in 1982) discovered six weeks after the beginning of term. In the other school both tests were administered but the students were not given the full time of 1 hour and 1 hour 15 minutes for A_4 and D_2 respectively. (As happened in 1981 for that school). Despite this lack of co-operation the data obtained are usefull.

Comments

1. The overall mean for D_2 and A_4 has not changed much and stands at 25.85, and 34.85 respectively. (Table IX). If we take into consideration comments made under 4 (that one National High School has a far superior intake than others) the overall mean would likely have been 24 or less for D_2).

2. The overall percentage of students who master the skills has remained constant at 48% (Table IX and Table XI)

3. The item-analysis for the D_2 test shows a number of constants Upward and downward trends appear for Decimals and Fractions respectively. The percentage of students who can do certain problems on decimals is increasing significantly for 8 problems. The percentrange of students who can do certain problems on Fractions is decreasing.

4. Whereas the overall mean for the D_2 and A_4 tests have remained constant over the years the quality of the intake varies now substatially from school to school. As is shown in Table X the Aiyura intake has always been of a poorer quality than that of the other National High School Schools. However in the Grade 12 Maths Examinations the Aiyura students performed at least as well and often better than the students of the other National High Schools. The Aiyura 1983 intake is of a much higher quality than the intake of any other institution at that level in the past. S.D.H.'s from Aiyura and Passam confirmed the difference in quality of the 1983 intakes.

A possible explanation for Aiyura's superior 1983 intake is that it has established a reputation for high standards. The Goroka Teaching Practice supervisor in East Sepik schools in 1982 became aware that the Regional Guidance officer was advising students that only theones with very top results should apply for Aiyura. Any others wishing to go to National High School should apply to the other schools.

From Table X one might infer that students have become aware of their chances for admission to further studies. Good students would prefer Aiyura to other institutions. Weaker students may get a better chance for further education when they apply for Goroka Teachers' College i.s.o. Waigani. I am still investigating this assumption.

I also have sent questionnaires to students in an attempt to find out why some students did so well (scoring over 40/50) in the D_2 test.

It should be noted that the mean for Technical Colleges and Teachers' Colleges are in the order of 14 and 24 for D_2 and A_4 respectively. Such colleges receive large numbers of upper passes and passess.

D_2 TEST

D_2 TIME: 1 hour and 15 minutes.

NAME: ..... AGE: .....

ANSWER ALL QUESTIONS. WRITE THE ANSWER ON THE DOTTED LINE. YOU MAY USE PAPER FOR ROUGH WORK. YOU MAY ALSO USE THE EMPTY SPACE ON THIS PAPER FOR ROUGH WORK. THERE ARE THREE PAGES ITH PROBLEMS. ALTOGETHER 50 PROBLEMS. IF YOU FIND SOME OF THE QUESTIONS A LITTLE BIT HARD, DO THE EASY ONES FIRST.

1. $5\frac{1}{4} - 2\frac{3}{4}$ = ... $15 - 8\frac{1}{4}$ = ...

$- 13 - 6\frac{2}{3}$ = ... $200 - 99\frac{3}{8}$ = ...

2. $8 \div \frac{1}{2}$ = ... $\frac{1}{2} \div 8$ = ...

$\frac{11}{3} \times$ ..... = 1 $5\frac{2}{5} \times$ ..... = 1

3. $2.16 \div 3$ = ... $0.16 \div 4$ = ...

$\frac{2.5}{0.5}$ = ... $12.35 \div 13$ = ...

$\frac{56}{800}$ = ... $8.464 \div 0.8$ = ...

4. Find a number bigger than 2.04 but smaller than 2.05 .....

Find a number bigger than 1 but smaller than 1.1 .....

Find a number bigger than 1.4 but smaller than 1.5 .....

Find a number bigger than -5 but smaller than -4.5 .....

5. 75% of a number is 60. What is the number? .....

$0.1 - 0.01$ = ... $10 + \frac{0}{10}$ = ...

$\frac{(123 \times 425) + (123 \times 575)}{246}$ = ...

$0.666666666666.. + \frac{1}{3}$ = ... $(987 \times 596) + (13 \times 596)$ = ...

$\frac{1}{R} = \frac{1}{3} + \frac{1}{6}$ What is the value of R ? .....

6. Find $(0.06)^2$ ..... $\sqrt{64 + 36}$ = ...

7. -When $2\frac{1}{2}a = 100$ What is 10a? .....

-2kg. of meat cost 5 Kina.
What is the cost of 5Kg.? .....

-A man works for 5 hours and gets paid 6 Kina.
What would he receive for 8 hours work, if he was paid at the same rate? .....

-David and Peter have to divide 4.80 shillings between them in the ratio 3:5. Peter gets ..... David gets .....

-A car travels from Goroka to Lae in 6 hours. The distance is 300 Km. How far will the car travel (at the same average speed) in 4 hours?

.....

8. Find in each case the missing number in the following tables:

Example:

X	Y
5	24
4	15
3	8
2	.

The missing number is 3 since the pattern is: Take x, square it, and then subtract 1.

$5^2 - 1 = 24$

$4^2 - 1 = 15$

$3^2 - 1 = 8$

Therefore $2^2 - 1 = 3$
The missing number is 3, put 3 on the dot.

X	Y
1	1
3	9
2	4
4	.

X	Y
2	5
3	10
4	17
5	.

X	Y
54	18
24	8
33	11
.	7

X	Y
0.1	0.09
1	0.99
8.9	8.89
0.7	.

X	Y
0.50	0.25
0.42	0.21
0.06	0.03
0.12	.

X	Y
16	12
8	6
10	$7\frac{1}{2}$
.	9

X	Y
3	1.80
5	3.00
6	3.60
4	.

X	Y
18	10
9	5
27	.
45	25

X	Y
5	13
7	17
4	11
6	•

X	Y
18	10
10	6
8	5
20	.

X	Y
5	1
8	4
15	11
7	.

9. What is the next number in the following number patterns?

17 8.50 4.25 .....

2 8 14 20 .....

0.4 0.2 0.1 0.05 .....

1.12 0.56 0.28 0.14 .....

1¼ 2½ 3¾ 4 .....

16 8 4 2 1 .....

When you have finished, check some some of the answers.

A4 TEST

TIME 1 HOUR

A_4

THIS PAGE IS FOR MARKING AND RECORDING. DO NOT USE THIS PAGE. Use all empty space for rough work.
THERE ARE 3 PAGES WITH PROBLEMS.

NAME: _____

AGE: _____

SCORE _____(out of 50)

Marking scheme: ONE MARK FOR EACH QUESTION RIGHT AND ZERO FOR THOSE WHICH ARE WRONG. Answer should be simplified!

The matrix is filled in by 1s and 0s depending on whether students are correct in ALL the questions in a particular section.

M A T R I X

	NATURALS	INTEGERS	FRACTIONS	DECIMALS
+				
-				
×				
÷				

After the matrix for individual students have been completed, totals are found and the percentage of students who master the particular skill is calculated.

A_4

ALL THE WORKING MUST BE SHOWN ON THE PAPER.

Use all empty space for rough work.

WORK AT A SPEED WHICH ENSURES THAT YOU WILL BE ACCURATE.

NATURALS

addition:
$$\begin{array}{r} 56769 \\ 8655 \\ + \ 97327 \\ \hline \end{array}$$

45689 + 49 =

Subtraction: 624 - 89 =

$$\begin{array}{r} 304 \\ - \ 58 \\ \hline \end{array} \qquad \begin{array}{r} 30000 \\ - \ 58 \\ \hline \end{array}$$

Multiplication:
$$\begin{array}{r} 45076 \\ \times \ 86 \\ \hline \end{array} \qquad \begin{array}{r} 6098 \\ \times \ 705 \\ \hline \end{array}$$

567 x 500 =

Division: 46926 ÷ 79 =

95)67355

INTEGERS

Addition -4 + (-6) =

(-15)+ 8 =

5 + (-9) =

Subtraction $5 - (-8) =$

$-8 - (-12) =$

$-5 - (3) =$

Multiplication $(-5) \times 3 =$

$6 \times (-7) =$

$(-12) \times (-3) =$

Division $(-45) \div 9 =$

$12 \div (-3) =$

$(-16) \div (-8) =$

FRACTIONS SIMPLIFY ALL ANSWERS e.g. $\frac{12}{15} = \frac{4}{5}$ and $\frac{5}{3} = 1\frac{2}{3}$

addition $\frac{2}{3} + \frac{7}{15} =$

$2\frac{1}{2} + 3\frac{3}{4} =$

$3\frac{2}{3} + \frac{1}{9} =$

Subtraction $\frac{2}{5} - \frac{1}{3} =$

$5\frac{3}{4} - 1\frac{1}{2} =$

$5\frac{1}{3} - 2\frac{2}{5} =$

$12 - 8\frac{2}{5} =$

Multiplication $\frac{2}{3} \times \frac{2}{3} =$

$\frac{2}{3} \times 3\frac{3}{4} =$

$3 \times 5\frac{3}{4} =$

$1\frac{1}{5} \times 1\frac{1}{4} \times 3\frac{1}{3} =$

Division $3 \div \frac{2}{3} =$

$\frac{4}{3} \div \frac{5}{6} =$

$3\frac{3}{4} \div 1\frac{1}{2} =$

$3\frac{3}{7} \div 3\frac{3}{5} =$

DECIMALS

Addition $0.2 + 0.3 + 0.9 =$

$0.68 + 2.708 =$

$908 + 0.089 + 8.3 =$

Subtraction $0.75 - 0.71 =$

$8.06 - 5.68 =$

$78.7 - 8.85 =$

Multiplication

$0.35 \times 0.35 =$

$$\begin{array}{r} 5.567 \\ \times\ 0.8 \\ \hline \end{array} \qquad \begin{array}{r} 0.067 \\ \times\ 0.78 \\ \hline \end{array}$$

Division $7 \div 100 =$

$4.5 \div 0.3 =$

$7.272 \div 0.9 =$

$0.45 \div 5 =$

TABLE IX (next page)

Means and standard deviations by institution of tests D_2 and A_4 for years 1978 to 1983.

Institutions listed are as follows:

A	- Kerevat N.H.S.	E	- Sogeri N.H.S.
B	- Aiyura N.H.S.	F	- Kabiufa High School
C	- Goroka Teachers' College	G	- U.P.N.G. Waigani Campus
D	- Passam N.H.S.	U	- P.N.G.U.T.

NOTE: The data shows that the means and standard deviations of both tests have remainded fairly constant at approximately 24 for D_2 and 33 for A_4 over the years.

TABLE IX

YEAR	INSTIT.	N	D_2 TEST		A_4 TEST	
			MEAN	S.D.	MEAN	S.D.
1978	A				32.25	
	B				36.86	6.21
	C				32.70	
	E				34.35	6.60
	TOTAL	705			33.90	7.73
1979	A		24.37	8.35	34.50	8.42
	E		24.35	8.55	34.35	7.98
	C		24.37	8.07	33.32	9.70
	TOTAL	463	24.02		34.05	8.30
1980	A		23.81	7.66	34.25	7.93
	B		22.14	8.02	29.10	9.54
	C		25.80	7.80	35.79	7.73
	D		22.40	8.60	36.86	7.49
	E		20.69	8.24	33.00	9.15
	TOTAL	828	23.33		33.96	8.18
1981	A		24.31	8.08	34.33	8.20
	B		23.72	7.69	30.56	8.83
	C		-	-	33.27	8.27
	D		25.96	8.69	35.65	8.33
	E		19.00	8.83	32.38	9.23
	TOTAL		23.38		33.38	8.40
1982	B		22.02	7.92	31.38	8.92
	C		26.51	9.12	-	-
	D		26.50	8.14	33.45	8.69
	E		16.89	8.48	31.14	8.42
	F		31.08	7.79	36.45	9.17
	G		18.71	7.69	33.83	8.76
	TOTAL		24.23		32.67	8.50
1983	B		29.40	7.98	34.40	8.11
	C		21.92	5.55	33.71	6.86
	D		23.30	8.80	34.30	8.90
	U		27.67	8.26	38.69	7.10
	G		29.79	9.13	-	-
	TOTAL		25.85		34.85	

TABLE X

Data shows the mean scores and standard deviations on the D_2 test of students who obtained a Distinction (D), Credit (C), Upper Pass (U.P.) or Pass (P) at year 10 level for mathematics.
NOTE: Within each category there seems little variation over the years.

YEAR	INSTITUTION		D	C	U.P.	P.
1979	Goroka T.C.	N	17	72	27	7
		Mean	31.47	25.55	19.11	14.14
		S.D.	7.24	7.49	7.14	5.54
1980	Passam N.H.S.	N	31	78	35	3
		Mean	31.55	23.82	16.20	13.00
		S.D.	5.26	7.90	6.91	3.00
1981	Passam N.H.S.	N	44	105	27	3
		Mean	33.09	25.93	18.30	11.67
		S.D.	7.66	6.96	6.10	3.79
	Aiyura N.H.S.	N	44	109	33	3
		Mean	28.73	23.37	19.36	13.33
		S.D.	6.87	7.15	6.18	4.51
	Kerevat N.H.S.	N	40	104	31	3
		Mean	30.43	24.10	19.00	12
		S.D.	8.05	7.74	5.23	5.57
1982	Passam N.H.S.	N	41	115	11	1
		Mean	33.24	25.00	16.45	16.00
		S.D.	8.35	6.64	2.50	-
	Aiyura N.H.S.	N	49	114	14	1
		Mean	28.96	20.09	14.79	18.00
		S.D.	6.77	6.61	5.49	-
	Sogeri N.H.S.	N	62	114	11	2
		Mean	33.95	23.79	15.64	12
		S.D.	6.99	7.71	4.03	1.41
	Goroka T.C.	N	6	42	26	1
		Mean	28.33	17.05	13.69	7.00
		S.D.	7.59	5.41	6.46	-
1983	Passam N.H.S.	N	57	99	27	4
		Mean	30.11	22.34	14.78	10.50
		S.D.	7.22	7.22	5.56	6.86
	Aiyura N.H.S.	N	54	109	15	-
		Mean	36.94	26.64	21.27	
		S.D.	6.36	6.10	4.86	
	Goroka (school leavers)	N	6	41	8	-
		Mean	28.67	21.63	15.25	
		S.D.	6.38	4.80	3.53	
	Waigani (school (leavers)	N	20	16	-	-
		Mean	31.15	24.80		
		S.D.	6.40	5.41		

TABLE XI

Percentage Matrix - The figures indicate the percentage of students who master the particular skills on the A_4 test. Mastery is defined as scoring full marks in a particular section.

(a) Overall percentage of students who mastered skills:

YEAR	%	N	INSTITUTIONS
1978	48	705	Sogeri, Goroka, Kerevat, Aiyura
1979	48	463	Sogeri, Goroka, Kerevat
1980	49	828	Sogeri, Goroka, Kerevat, Aiyura, Passam
1981	47	669	Sogeri, Goroka, Kerevat, Aiyura
1982	43	605	Goroka, Aiyura, Passam, Waigani
1983	48	456	Goroka, Aiyura, Passam, Waigani

(b) Percentage Matrices:

Code N - Naturals section Fr - Fraction section
I - Integer section D - Decimal section

1978 A_4

	N	I	Fr	D
+	81	53	48	61
-	85	26	28	61
x	52	71	27	35
÷	41	61	30	12

1979 A_4

	N	I	Fr	D
+	76	51	40	60
-	81	24	27	60
x	51	71	54	42
÷	40	57	29	17

1980 A_4

	N	I	Fr	D
+	75	59	46	60
-	84	34	31	55
x	51	80	29	35
÷	35	71	32	15

A_4 1981

	N	I	Fr	D
+	81	57	41	60
-	84	22	30	56
x	49	72	26	32
÷	40	60	29	13

1982 A_4

	N	I	Fr	D
+	79	52	43	54
-	62	24	28	59
x	45	65	25	29
÷	31	56	28	11

1983 A_4

	N	I	Fr	D
+	83	35	53	54
-	82	36	32	55
x	48	78	30	39
÷	37	69	31	7

TABLE XII

Item Analysis of Test D_2:

Facility and rank order are based on results for 1982. Data from 1981 and 1983 are added for comparison. Note however, that results from different institutions were obtained in the three years.

The institutions which returned scripts are shown separately in the table and are listed as follows:

G - U.P.N.G., Waigani Campus

D - Passam N.H.S.

C - Goroka Teachers' College

B - Aiyura N.H.S.

E - Sogeri N.H.S.

*(F) - Kabiufa High School

*(F) was not included in overall results.

ITEM ANALYSIS FOR THE D_2 TEST

RANK	QUESTION	TOTAL SCORE (OUT OF 784	% FOR 1982	GCE (F)	OVERALL % 1982	OVERALL % 1981 1983	COMMENTS
1	Pattern 2 8 14 20	663	85 90 82 83 83 (71)		85	79 87	The percentage of studen who scared the right answer is very consist-ant.
2	Pattern $y = x^2 + 1$	659	86 92 86 81 73 (79)		84	86 81	Similar to the example given $y = x^2 - 1$
3	Pattern $y = x^2$	645	86 89 84 80 70 (71)		882	82 83	Similar to the example given
4	0.1 - 0.01	599	80 84 74 74 68 (50)		76	72 78	Popular answers: 0.1, 0.99, 0.11, 0.9

ITEM ANALYSIS FOR THE D_2 TEST

RANK	QUESTION	TOTAL SCORE (OUT OF 784)	SCORE (F)*	OVERALL 1982	OVERALL 1981 1983	COMMENTS
5	2.16 ÷ 3 =	589	83 81 74 73 68 (50)	75	63 80	Popular answers: 7.2 .702, 72
6	Preportionality table x = 3y	577	84 79 76 69 (58)	74	68 69	
7	2Kg. of meat costs 5 Kina. What is the cost of 5 Kg.?	548	77 79 76 74 63 * (79)	70	71 80	*School F does a lot of practical work in agriculture. They actually run a trade store.
8	Find a number bigger than 1 but smaller than 1.1.	507	71 66 64 65 57 (46)	65	52 73	

RANK	ITEM ANALYSIS FOR THE D_2 TEST QUESTION	TOTAL SCORE (OUT OF 784)	D C B (A)*	OVER 1982	COMMENT
9	8 ÷ ½ =	475	56 67 63 69 39 (42)	61 → 65 59	A popular answer: 4
9	Pattern 16 8 4 2 1	481	59 72 63 60 47 (58)	61 → 48 62	Popular answers: 0, 0.5 0.0, -1, 0.1, 32.
10	Pattern y = x - 4	472	66 72 67 51 40 (50)	60 → 56 57	
11	0.16 ÷ 4 =	462	64 62 62 58 47 (25)	59 → 57 56	A popular answer: 0.4

ITEM ANALYSIS FOR THE D_2 TEST

RANK	QUESTION	TOTAL SCORE (OUT OF 784)	% G D C B E (F)*	OVERALL % 1982	OVERALL % 1981 1983	COMMENTS
11	12.35 ÷ 13 =	459	62 59 70 54 43 (38)	59	63 63	Popular answers: 9.5, 95, 0.941.
12	Proportionality table y = x/2	443	59 58 73 46 42 (46)	57	48 54	Popular answers: 0.24 0.6 (many).
12	A car Travels from Goroka to Lae. The distance is 300 km. How far will the car travel (at the same average speed) in 4 hours?	439	59 64 61 56 44 (67)*	56	57 66	*See rank 7.
13	Proportionality table y = 0.60 x	422	54 66 63 42 40 (29)	57	48 52	

RANK	ITEM ANALYSIS FOR THE D_2 TEST QUESTION	TOTAL SCORE (OUT OF 784)	G D C B A E (F)*	OVERALL 1982	 1981 1983	COMMENTS
14	$\frac{1}{2} \div 8 =$	416	30 58 60 52 34 (46)	53	51 53	A popular answer: $\frac{1}{4}$ or 4
14	Pattern $y = 2x + 3$	415	59 65 53 45 34 (46)	53	49 54	
15	$\frac{11}{3} \times \ldots\ldots\ldots = 1$	407	55 56 56 48 43 (46)	52	61 56	Popular answer: 3, 33.
15	$\frac{56}{800}$	406	62 56 86 48 32 (30)	52	42 55	Popular answers: 007, 0.007, 0.7, 00.07.

ITEM ANALYSIS FOR THE D_2 TEST

RANK	QUESTION	TOTAL SCORE (OUT OF 784)	% G D C B E (F)*	OVERALL % 1982	OVERALL % 1981 1983	COMMENTS
15	$\sqrt{64 + 36} =$	406	50 63 53 49 38 (71)	52	43 57	Popular answers: 10^2, 50, 14, 100, 10000, 1.4.
16	$\frac{2.5}{0.5} =$	403	52 58 54 51 40 (42)	51	57 56	Popular answers: 0.5, 0.05
16	$(0.06)^2 =$	399	**53** 65 54 48 28 (35)	51	52 50	Popular answers: 0.03, 0.12, 0.036, 0.012, 000.36, 000.36 etc.
17	$2\frac{1}{2}a = 100$ find $10a$	392	48 44 49 44 38 (42)	50	41 44	

ITEM ANALYSIS FOR THE D_2 TEST

RANK	QUESTION	TOTAL SCORE (OUT OF [illegible])	D / P*	OVERALL 1982	OVERALL 1981 1983	COMMENTS
			60 52 54			
18	Find a number bigger than 1.4 but smaller than 1.5	385	48 29 (38)	49	43 56	Some students write: $1.4\frac{1}{2}$
			59 51 57			
19	David and Peter have to divide 4.80 shillings between them in the ratio 3 : 5. Peter gets ..... David gets	377	38 30 (46)	48	36 34	
			52 58 52			
19	Pattern 0.4 0.2 0.1 0.05	376	48 23 (25)	48	44 53	A number of students used a wrong notation ($0.02\frac{1}{2}$)
			55 54 49			
20	$5\frac{1}{4} - 2\frac{3}{4} =$	368	44 31 (33)	46	60 49	Many students get 10 as an answer. They just don't know anymore what they are doing.

ITEM ANALYSIS FOR THE D_2 TEST

RANK	QUESTION	TOTAL SCORE (OUT OF 764)	GROUP (F)	OVERALL 1972	OVERALL 1973	
20	$15 - 8\frac{1}{4} =$	362	53 47 55 47 23 (13)	46	45 48	Students use clumsy methods or wrong methods. e.g. $\frac{60 - 33}{4}$; $15-8\frac{1}{4} = 7\frac{1}{4}$ (many!)
21	Find a number bigger than 2.04 but smaller than 2.05	352	54 47 51 44 24 (29)	45	38 53	
21	Pattern 17 8.50 4.25	349	48 51 47 43 26 (42)	45	47 49	A number of students used the wrong notation $2.12\frac{1}{2}$ Popular answers: 2.5, 2.05 2.12 + 1
22	$\frac{1}{R} = \frac{1}{3} + \frac{1}{6}$ What is the value of R?	326	41 52 38 39 21 (21)	42	45 44	Popular answers: ½, 9,

ANALYSIS FOR THE D. TEST

RANK	QUESTION	TOTAL SCORE OUT OF 784		OVERALL % FOR 1981 1983		
23	$10 + \frac{0}{10} =$	319	44 51 42 33 32 (38)	41	48 43	1, $\frac{10}{11}$, $\frac{10}{10}$, $\frac{0}{20}$, 100 etc
24	8.464 ÷ 0.8	317	49 44 49 33 25 (8)	40	31 38	The answers for this question indicate that decimal point, place value and in particular the algorithm are badly understand and badly taught. answers: 1058, 1.059, 15.8, 1.58, 0.0158 etc.
25	Pattern 1.12 0.56 0.28 0.14	309	48 52 56 42 35 (21)	39	45 49	A popular answer: 0.7 (see 0.6 (rank 12).
26	$5\frac{2}{5}$ x ..... = 1	294	43 49 40 34 20 (38)	38	40 41	Popular answers $5\frac{2}{5}$ and 2.5

	ITEM ANALYSIS FOR THE QUESTION	TOTAL SCORE (OUT OF 784)	G D C % B E (F)*	OVERALL 1982	OVERALL 1981 1982	COMMENTS
27	Proportionality $y = \frac{x}{9} . 5$	288	33 32 39 28 24 (17)	37	26 26	
28	75% of a number is 60. What is the number?	280	38 33 40 31 19 (13)	36	21 33	A very popular answer: 4
29	Pattern 1¼ 2½ 3¾ 4	266	34 36 40 20 32 (21)	34	31 34	
30	Pattern subtract 0.01	244	36 32 43 23 18 (21)	31	25 27	A popular answer: 0.79

ITEM ANALYSIS FOR THE D_2 TEST

RANK	QUESTION	TOTAL SCORE (OUT OF 784)	A B C D E (F)*	OVERALL % 1982	OVERALL % 1983	COMMENTS
30	Pattern $y = \frac{x}{2} + 1$	240	33 36 41 19 20 (21)	31	25 26	*(no error)
31	$200 - 99\frac{3}{8} =$	225	30 33 40 26 8 (4)	29	25 30	(See comment on rank Students' method break down. Popular answers: $101\frac{5}{8}$, $\frac{80}{8}$
32	$\frac{(123 \times 425) + (123 \times 575)}{246}$	216	31 30 35 22 18 (17)	28	30 26	Although the distribut property is taught in Grade 10 very few students are aware of it.
33	$(987 \times 596) + (13 \times 596)$	169	30 21 26 19 12 (25)	22	28 26	(See comment under ran 32.)

ANALYSIS FOR THE D_2 TES.

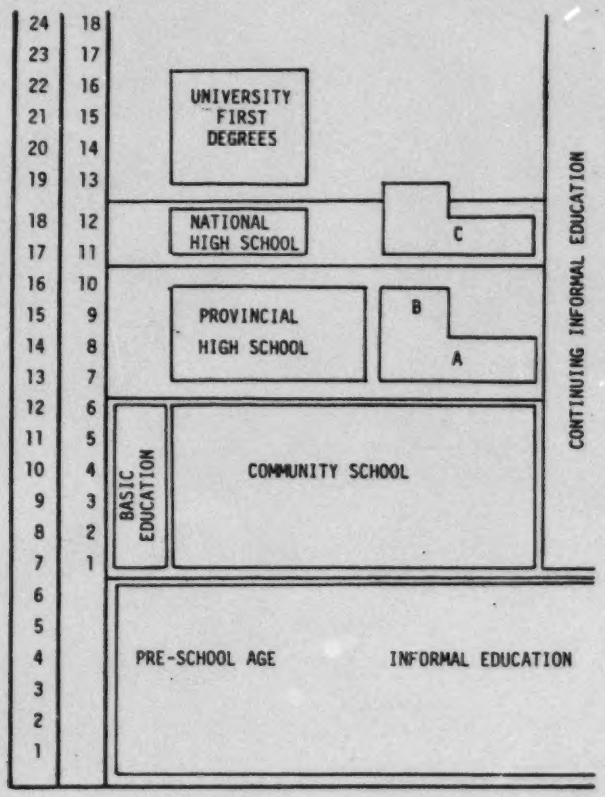
	QUESTION	TOTAL (OUT OF)	CODE (F)*	OVERALL % 1982	OVERALL % 1981 1983	COMMENTS
34	Find a number bigger than -5 but smaller than - 4.5	159	27 19 29 17 11 (13)	20	13 19	Integers are badly under stood.
35	Proportionality table $y = x/3 \cdot 4$	147	14 23 29 11 13 (17)	19	13 16	See Jones' findings (E.R.U. report 6, 1973.
3	$0.666666666666 + \frac{1}{3} =$	149	18 26 22 15 11 (4)	19	14 22	Some popular answers: 0.222222222 0.766666666 0.933333333 etc.
36	$-13 - 6\frac{2}{3} =$	138	21 15 31 11 8 (8)	18	15 14	See comment under rank 34.

DECISION MAKING AND THE NATIONAL HIGH SCHOOL CURRICULUM

John Bailey Department of Education.

INTRODUCTION:

The four National High Schools (Aiyura, Keravat, Passam and Sogeri) are key institutions in the Papua New Guinea Education System. These Grade 11 and 12 schools provide a major route to University and other tertiary institutions (Fig 1).



A = Non-formal programmes
B = Vocational centres
C = Teacher education, technical training, professional training, university diplomas

Figure 1 Structure of the Papua New Guinea Education System

The role of the National High Schools (NHS) was described in 1977 (Fig 2). The first of these aims emphasise that this is a major educational route of the nation's academic elite. Later down in the list the aims indicate in very broad terms how the curriculum should be organised.

ROLE OF NATIONAL HIGH SCHOOLS

To aid students in a successful transition from secondary to tertiary education or training for positions in private enterprise and/or the Public Service.

To aid the intellectual development of selected students by providing a further two years of secondary education.

To develop further self discipline and each individual's sense of responsibility.

To foster student pride in traditional heritage.

To encourage the study of, and involvement in (through service), of the local and wider community.

To provide opportunities for the development of leadership potential both inside and outside the classroom.

To provide courses in the five areas of English, Mathematics, Social Science, Science and Expressive Arts.

To increase student's political awareness and understanding of social issues.

(Department of Education 1977)

Figure 2 Role of National High Schools

What I hope to do in this paper is to describe how the National Institutions Division of the national Education System reponsible for National High Scools operates and makes decisions about the curriculum. Before I begin I would like to clarify a few points.

1. This paper is obviously not about Mathematics Education per se but about the context which influences what is taught in Mathematics.

2. Although included in this section of the conference on post Grade 10 mathematics this paper is based on a very small aspect of research leading towards an illuminative case study report.

3. I have utilised an alternative model to the traditional experimental model. This model is conceptualised as a RESEARCH CYCLE (Fig 3). The focus is on trying to understand the curriculum in this context; what factors have influenced the development of the curriculum and who 'controls' it.

4. The research has concentrated on the development of science education but I hope that some of the general ideas will be of interest.

5. This paper does not necessarily describe the official views of the situation and the conclusions are my own.

CURRICULUM OFFICER AS RESEARCHER:

Two years ago I was involved in an attempt to bring about a major change in the National High School curriculum (Bailey 1983). This involvement led me to view Stenhouse's idea of curriculum officer as researcher, rather than that of a man with a mission, or administrator/bureaucrat, with some sympathy (Stenhouse 1975).

Certain aspects of the situation I found myself in led to me feeling dissatisfied with the curriculum approach being utilised. The choice of curriculum development model (a behavioural objectives, system approach) seemed to ignore the organisational structure of the sub-system, as I understood it, and the reality of the internal operating systems of National High Schools. At that stage I thought that in the National High Schools the curriculum was developing on an ad hoc basis, decisions being made, making small changes, that were tried and rejected in a rather haphazard manner. The implications of these decisions often did not seem to be considered. This appeared on the surface to be a genuine evolutionary situation with mutants appearing, some beneficial, some detrimental, some surviving, some not. I wanted to understand this situation and this meant researching the system, historically and currently.

The curriculum administrative organisation (Fig 5) is complex with considerable school input at the decision-making level (principals, subject department heads and teachers). The administrative structure sets the main framework for my attempt to understand the situation. The focus then is the context rather than the learning situation resulting in an analysis of how the syllabuses were derived, the content of the syllabus as recorded in the resource materials and finally interpretation in the classroom. The whole cycle is almost completed for Science but not for Mathematics but the methodology could be of interest.

THE RESEARCH CYCLE.

From the beginning I was aware that the type of understanding I was seeking could not be gleaned by using an empiricist-positivist approach utilizing the traditional educational experiment seeking to refute hypotheses. I needed an alternative approach to seeking knowledge. If the research was not to be framed within the classic scientific model, what other models were available. During the first exploratory phase two books had a considerable influence on my meanderings.

The first was the new paradigm resource book 'Human Inquiry' editted by Reason and Rowan (1981) and the second the structuralist view of curriculum innovations in developing countries 'Solving Educational Problems' By Havelock and Huberman (1978). It is from the former that I have utilized a model conceptualised as a Research Cycle (Fig. 3)

The model was put foreward to reconcile the traditional scientific research paradigm and the new collaborative, participatory and co-operative research styles based on humanistic psychology (Rowan 1981). Rowan utilised the Research Cycle to encompass and compare the various approaches. I have used the cycle more at the dialectic end of the spectrum to organise and explain my activity while compiling the case study of the National High Schools Curriculum.

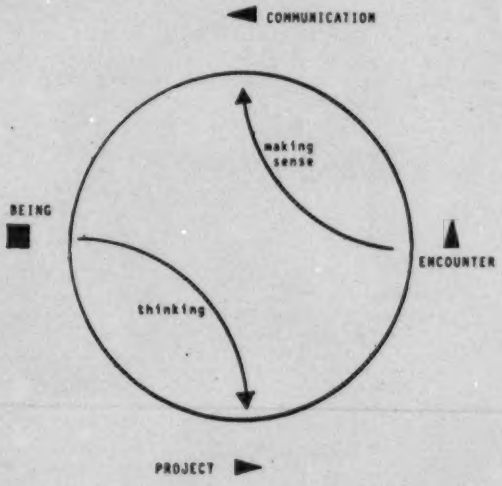


Figure 3: THE RESEARCH CYCLE

UTILISING THE RESEARCH CYCLE

BEING: I started by feeling dissatisfied with my existing practise and a need for change. A problem had arisen. This provided the initial energy that led to the decision to research the context of my job. This stage of the cycle I have briefly described in'Curriculum Officer as Researcher.'

Thinking: How to solve the problem. What does 'curriculum' mean to myself and others in this situation? What is a 'curriculum officer'? This phase concentrates on the collecting of easily available information What others know already: conversations, phone calls, meetings. Ideas start flying around. A contradiction arises between needing more information and having too much already.

PROJECT: At a certain stage disatisfaction again sets in. Thinking is not enough. Action is needed. Others need to be involved. Risks have to be taken. It essentially involves plans and decisions. It involves a contradiction 'plans should be adequate' versus 'no plan can be perfect'. The decisions were made and the following plan drawn up.

Fig 4 PROJECT: THE CASE STUDY PLAN

To guide the drawing up of the plan I decided to seek answers to four central questions.

1. What is 'curriculum' in this situation?
2. What 'factors' have shaped the current curriculum?
3. Who 'controls' the curriculum and does this 'control' result in the mostsuitable curriculum for the country's needs?
4. Can the current curriculum be taught to reflect the way it is written down?

To approach these questions I focused on the idea of 'control' but I may mean influence. To direct my encounter with the National High School curriculum I chose three levels and my intention was to use various methods.

Level A	Administration
	Documentary analysis especially of minutes of Board of Studies and Syllabus Advisory Committees with associated papers; and informal interviews.
Level B	The Written Curriculum
	Content analysis of syllabuses using as a frame of reference their own stated aims and objectives.
	Analysis of materials produced by the Syllabus Advisory Committee.
	Analysis of examination papers.
	Reading of relevant papers.
	Informal interviews.
Level C	Teachers and Students in the Classroom
	Annotation of trial materials.
	Feedback questionnaires.
	Interviews with teachers and selected students.
	Videotape of lessons.
	Observation of lessons.

ENCOUNTER: Again at a certain point thinking about investigating and drawing up plans is not enough it becomes necessary to begin and the encounter with the information sources starts. At the administrative level the encounter has mainly been with minutes of meetings and background papers enriched however by often knowing and talking to the people involved, and knowing the end result of each of the debates.

The written curriculum was mainly encountered by an analysis of the written constraints put on the classroom teacher (whether they allow themselves to be constrained is another matter). Finally there was the encounter with the classroom.

Making sense: At each of these levels the encounter phase was followed by making sense of the information that I had aquired. However analysis is not enough synthesis and evaluation must follow. I think I understand the situation in a clearer way. But do I. I must tell somebody to identify mistakes and errors of interpretation and perception. I must communicate.

COMMUNICATION: This is the stage where the new information and ideas have become part of my new accommodation to reality. To how I percieve my job (or the research context focused on). The ideas and information need to be in a form that can be communicated. Obviously this is the phase we are participating in now.

WHO DECIDES ON THE MATHS TAUGHT TO NATIONAL HIGH SCHOOL STUDENTS?

This is not a simple question. There are a number of possible candidates and their relationships are complex.

Department of Education (Secretary), National Institutions Division, Board of Studies, Mathematics Syllabus Advisory Committee, National High Schools, Mathematics Department of National High Schools, the Teachers, the Students. But of course all these elements are made up of individuals who can have complex relationships with some or all of the elements and other involved individuals.

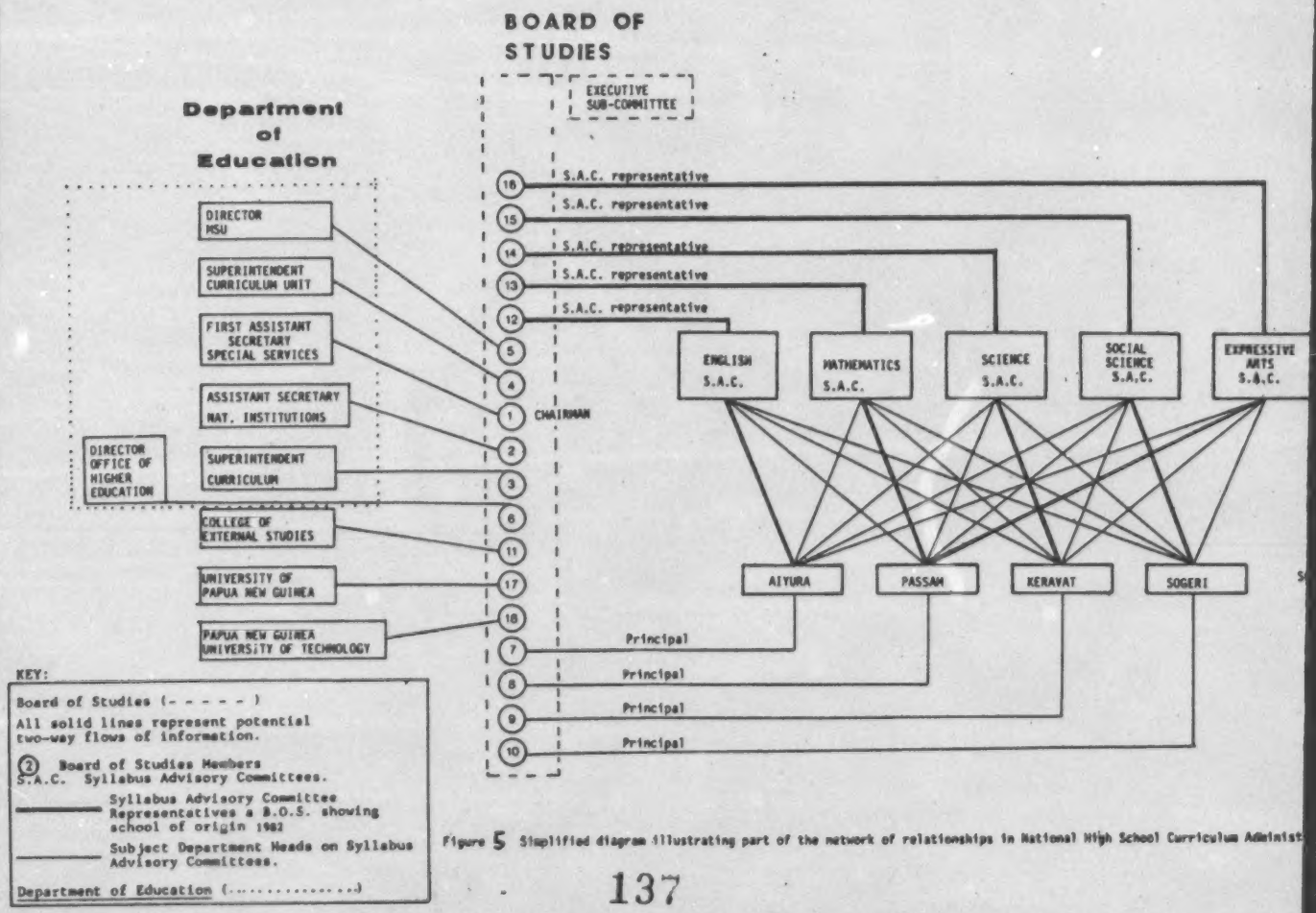


Figure 5 Simplified diagram illustrating part of the network of relationships in National High School Curriculum Administ

The central element is the National High School Board of Studies. This Board advises the Secretary for Education on standards, curriculum and related matters. Recommendations of the Board (including syllabuses) do not become policy until approved by the Secretary for Education. Another body, the National High School Committee, a sub-committee of the National Education Board exists to advise on matters of planning, enrolment, finance etc.

The Board of Studies is comprised of eighteen members, ten of which are employed in schools (five principals and five subject department heads), of the remaining nine, two are university representatives while seven are personnel from different Department of Education sections (Fig 4).

The crucial element for Mathematics Education is the National High School Mathematics Syllabus Advisory Committee (MSAC). This committee is comprised of the Mathematics Subject Department Heads, University representatives and other interested people. In 1976 the name Senior High School was changed to National High School and I have taken that date as the start of the evolution of the NHS mathematics curriculum. In that year the committee was given the role of developing common courses for the National High Schools.

The role of the MSAC has remained constant since its inception. However there has been changes in responsibilities of the members mainly due to the creation of the National Institutions Division and the withdrawal of the curriculum unit from a leadership role although a representative remains on the committee. The committee is mainly concerned with development and production of syllabuses, examinations and assessment patterns, and the development and production of teaching materials.

DECISIONS OF THE MATHEMATICS SYLLABUS ADVISORY COMMITTEE AND THE BOARD OF STUDIES:

Development of The Mathematics Curriculum In National High Schools:

Syllabuses: Although existing National High Schools had syllabi prior to 1976 these were developed within each school. The development of common syllabi originates from 1976. Two years later a Trial Grade 11 and Grade 12 syllabus was published by the Department of Education in 1978. As always the trial indicated that the syllabus would need revision.

This trial syllabus was really three syllabi: Grade 11, Grade 12 Major, Grade 12 Minor. The revised Grade 12 Major course was adopted in 1981, the revised Grade 11 in 1982 and the revised Grade 12 Minor course in 1983. The level of teacher involvement was greater in the revised syllabi.

The course revisions included no major changes but were more concerned with re-organising the content and clarifying the meaning of the document so that it could be more useful as a teaching framework. Examples of the type of change to Grade 12 Major course are.:

1. Section 1.2. to include the Probability Section from Grade 11, increasing time from 5 to 6 weeks

2. Previous topic 12.3 Social Maths to be divided between Algebra (use of formula) which gains 1 weekand Sequences and Series (compound interest) which also gains 1 week.

3. Rates and Changes altered its name to Introduction to Calculus but content to remain about the same.

4. There were also changes to the computing section which I will talk about later.

NATIONAL HIGH SCHOOL MATHEMATICS COURSES Fig 6

GRADE 11 COURSE

Teaching Time: 36 weeks

Testing + Revision: 4 weeks

Six 50 minute lessons per week

BASIC NUMERACY	5 weeks
BASIC GEOMETRY	3 weeks
RATIO AND PROPORTION	3 weeks
MEASUREMENT	3 weeks
ALGEBRA	10 weeks
GRAPHS	3 weeks
STATISTICS	2 weeks
TRIGONOMETRY	4 weeks
SOCIAL MATHEMATICS	3 weeks

Students are expected not to use calculators for the majority of the work.

GRADE 12 MAJOR COURSE

Teaching Time: 32 weeks

Testing + Revision: 8 weeks

Eight 50 minute lessons per week

PROBABILITY	6 weeks
COMPUTER APPRECIATION	1 weeks
ALGEBRA	7 weeks
GRAPHS + CURVES	3 weeks
SEQUENCES + SERIES	3 weeks
TRIGONOMETRY	4 weeks
VECTORS	2 weeks
ANALYTIC GEOMETRY	3 weeks
INTRODUCTION TO CALCULUS	3 weeks

GRADE 12 MINOR COURSE

Teaching Time: 32 weeks

Testing + Revision: 4 weeks

Four 50 minute lessons per week

SOCIAL MATHEMATICS	8 weeks
STATISTICS	8 weeks
MEASUREMENT + APPLIED TRIGONOMETRY	9 weeks
ALGEBRA	7 weeks

Students are expected to use a calculator.

TEACHING ORDER IS NOT PRESCRIBED ON THE SYLLABI

' The syllabuses are traditional, with a heavy emphasis on algebra and graphs, simple trigonometry and statistics. Special concepts are largely ignored. Most of the algebra, trigonometry and vectors is pure' (Goldsmith 1981).

Examinations, Assessment and Moderation

With the advent of a common syllabus, common examinations were developed. In 1979 a common examination taken by students in all four schools was based on questions submitted by the schools. From 1980 the papers have been set externally with heavy involvement of the Mathematics Education Centre, in the process.

1980 saw a trial moderation which was deemed to be successful This moderation distributed the 5% A, 25% B, 40% C, 25% D, and 5% E through the national population rather than through the in-school population and was deemed successful.

The trial moderation established the moderating system and was fully implemented in 1981. The examination and an internal mark are combined in a one to one ratio to produce the final mark, a standardised score. The Subject Department Heads are still involved in the system as markers co-ordinated by the director of MEC. The responsibility for examination papers still lies with the MSAC who draw up a specification grid and other rules.

Materials:

Calculators: The MSAC lays down regulations for the use of calculators. Until recently a specific calculator was required to be purchased for use in schools. For the future the following criteria have been laid down: same logic as the TI30, all standard operations, standard trigonometry inverse trigonometry, power functions and radians.

Teaching Notes: Sets of teaching notes are being developed. This was started in 1981. It is intended that they should indicate depth and breadth of treatment of syllabus topics. No specific format is laid down, style depends on individual NHS teachers who write them. To aid the development a National Institutions Division curriculum officer has set up two expandable files known as the National High Schools Mathematics

Teaching Material files. The files contain material approved by MSAC:

FILE 1: A. General Teaching Notes
B. Examples
FILE 2: C. Test/Examination Material
D. List of Resources/Equipment needs in Each Topic.

The curriculum officer also operates a pending file into which committee members can put material which has been circulated to all other committee members beforehand for inspection, before seeking approval for inclusion in the official files at the next full committee meeting.

Textbook: The committee has approved in principle the preparation of a textbook for the Grade 12 Major course this is. This is being prepared by an experienced SDH.

Equipment: A booklet has been produced listing Structures, Aids and Equipment useful in the Maths courses.

Computing in the Mathematics Syllabus:

At this stage I would like to briefly comment about Computing in the Mathematics Syllabi and return to it again later in the discussion. In the 1978 set of syllabuses a Computer Studies course was to be found in both the Major and Minor courses. The topic was allocated 3 weeks in the Major course and four weeks in the Minor course. (In the Minor course the content was also reduced) In the 1980 revised Major Syllabus the course was reduced to a one week awareness course, and the revised minor course dropped the topic completely.

The Goldsmith Report (1981)

The call for papers for this conference included 'theoretical position papers on such topics as methods of reporting research to the educational decision makers'. I would like to suggest that the Goldsmith report is an example of a good report in the descriptive evaluation field. It is short, to the point and emminently readable. The Goldsmith Report 'Evaluation of the Grade 11 and 12 Mathematics Curriculum' was produced as the result of a five week visit at the end of 1980. The twenty page

report, included seven pages of general recommendations, criteria for syllabus construction and revision, as well as a draft teaching plan. Some of the recommendations are in the process of being implemented for they focused current ideas from the people involved in Mathematics Education into one document. Computing has been reduced in the curriculum, development of teachers notes, worksheets and textbooks picked up after a curriculum officer was identified to act as editor and co-ordinator and the assessment and examination system has become more regulated.

COMMENT

Now I realise that many of you have been more intimately involved with the development of Mathematics Education in the National High Schools than I have. My persepective is from the standpoint of a curriculum officer that has lived in a National High School for three years with no teaching committment, but a general brief across the curriculum of the national high schools.

In my description of the research cycle I pointed out that one way of validating ones own views is to communicate them and see how concordant they are with the perspectives of others. However before giving you the chance I would like to indicate how I perceive the decision--making system.

1. Curriculum is not just what happens in classrooms and schools. It is influenced by various formal groupings in the system and within these the views of particular dominant personalities can have strong effects on the content (but less so on the classroom teaching as this is harder or impossible to control by curriculum documents)

2. Before the creation of the National Institutions Division the style of curriculum development was probably system based directed from the centre. With the transference of the chairmanship of the MSAC to one of the SDH's and the change of role of the curriculum unit a more school-based approach developed. In this particular situation both could be seen as forms of democratic curriculum development.

3. Like any organisational body both the Board of Studies and the Maths Syllabus Advisory Committee has its strengths and weaknesses. If I take the weaknesses first. Professional and institutional conservatism is a well recorded phenomena and because of the constituted membership of the committees they reflect this conservatism. If it were necessary to bring about any fundamental change in the syllabus documents intended to change teaching approaches this conservatism

would need to be considered. With the current organisation and representation the chances of any curriculum development approach that was not evolutionary would be unlikely to succeed. In a large system where ideas can be tried out in particular schools without putting the assessment system out of equilibrium the position would be easier. Documented cases of this conservatism is shown in the response to Design & Technology, Computer Studies as a separate course and Japanese. All of which were resisted.

4. The MSAC conservatism is not an irrational and negative activity and I do not mean to suggest that it is, for it is the result of a particular world-view. The devolution of the Computer Studies courses are a case in point. I would have prefered to see computing firmly within another niche first. I t has been suggested to me that Computer Studies was rarely taught, if true, this is not unusual in evolving syllabuses some topics always take longer to implement than others, especially if they lay at the edge of the teaching experience of the staff. In retrospect they were probably put into the syllabus too early, before there was staff with an adequate background, before there was suitable hardware to make it interesting. However by the time the topic was removed from Minor Mathematics staff were qualified, interested and the hardware was beginning to appear in the schools. Admittedly the content of the course would probably have needed drastic revision. In one way the minor course with its emphasis on social mathematics would have been and still is an excellent place for some aspect of computing education.

5. The strengths of the two committees is based on the unique involvement of the teaching staff, administration and the universities in a continuous dialogue. The heavy involvement of the SDH's (from all the schools) on the MSAC and the Principals on the Board of Studies leads to an environment that increases the likelihood of the implementation of any decisions.

6. The evolution of the mathematics education in national high schools of PNG has in some ways been a normative exercise in getting current expensive experience documented. This has resulted in some fulfillment of Stenhouse's minimum requirement that a curriculum should provide a basis for planning a course, studying it empirically and considering the grounds for justifying it" (Stenhouse 1975).

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First Year University Students' Mathematical Performance On the 1982 End of Course Examination

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ABSTRACT: This paper extends tha analysis of earlier work of the Mathematics Education Centre on the mathematical performance on examinations of year 11 and year 12 school students.

Comparisons will be made between male and female students, between ex-Preliminary year students (Preliminary year is a one-year university course which is alternative to the two-year grade 11 and 12 high school course) and ex-grade 12 students.

Further comparisons will be made between overall performance and performance on particular mathematics topics, and between analytic mathematics skills and spatial ability mathematics skills.

Mastery levels will be considered for the exam as a whole, for separate topics, and for individual items.

Some comparison will be made between this 1982 university performance and the preceding 1981 grade 12 performance.

Mathematics - A preparation for study in Engineering and Applied Science.

M.J. Priest

I wish in this paper, to suggest some of the difficulties that are encountered in applying the mathematical skills of PNG students to the solution of engineering problems, and their success in achieving objectives. Although a relatively small overall proportion of National High School students move into formal tertiary engineering study, it is likely that many of the comments presented here are applicable to related applied science areas, and other levels of vocational education.

In suggesting some of these difficulties I cannot claim to know what the immediate answers will be. It will be satisfying however if we can stimulate some discussion of the nature and scope of the problems encountered. This may allow for some of the factors involved to be considered at times when the Mathematics curricula and teaching strategies are under review.

A major problem is one of language. I can only feel sympathy for students learning in their 2nd or 3rd language. An example may be useful here. Most of our final year B.E.E.L. students were recently able to prove that

$$\nabla \times \nabla \times \underline{E} = \nabla(\nabla \cdot \underline{E}) - \nabla^2 \underline{E}$$

But the same students will wallow helplessly in preparing a project report including analysis. Some would say that we can do very little to relieve this problem. However it may often be more useful to present problems as written

statements requiring language interpretation rather than in the form of a purely numerical or algebraic statement. I understand that this is done, certainly at the year 12 examination level, but I am less aware of its widespread application. It may also be useful to continue formal English throughout the course.

Engineering (and many of the applied sciences) require a large degree of evaluation skills. It is necessary to identify the problem, analyse, and evaluate. This overall process is, by the nature of its application criteria tested. - Will the bridge fall down?, Does the equipment meet specifications?, Will the transistor burn out? A 50% record on bridges may possibly have career implications.

The Institutions of Engineering and other bodies make extensive use of Blooms Taxonomy of Behavioural objectives, in their evaluation and accreditation of courses of study. Objectives in Engineering courses include terms such as:

Demonstrate, Evaluate, Design, Apply.

Behavioural objectives of Engineering include:

(a) A high level of abstract, manipulative and perceptive thinking.

(b) The ability to apply learned skills to new situations.

(c) The ability to apply knowledge and skills to problem solving in practical situations.

These objectives cannot be met by an assessment system that allows progress with a 50% competancy in the mechanics or syntax of the mathematical language. Application and continual cross referencing are essential to the needs of Engineering.

Students in PNG appear to have evolved survival techniques to an unusual extent. These include extensive use of rote learning and problem replication. These techniques are not compatible with professional objectives of judgement and evaluation.

It would appear to be more suitable to identify a curriculum with less analysis, to teach this on an applied analysis strategy, and to perform assessment based on mastery tests of individual units. This would involve not only a reduction in the amount of analysis, but also an identification of essential analytical and numerical techniques. A close co-ordination with other parts of the curriculum in implementing tuition for mastery in these essential techniques would then become necessary. This requires a close liason between Departments on required analytical and numerical skills, and their location in the syllabus.

It is desirable that the mathematical skills of a student who aspires to succeed in an Engineering course should enable him to put together a complex whole by the union of elements, and to form concepts from the processing of theories. But many of our students at present exhibit only an ability to recite from memory, or to reproduce a standard solution. Even more seriously, these students appear to consider that this is what is required for academic success. These survival techniques may be likened to an endemic disease. They are so widely encountered that there is a danger of them becoming part of the Educational structure.

There is a worry in Engineering Departments about the failure to observe improvement in meeting objectives, as our courses, and our students develop into an era when the entry level is higher and the exposure to Western technology significantly longer. A working party on Foundation Year Engineering and Physics was established in 1982. Concern had been expressed about the mathematical performance in later year engineering courses, especially considering that 60% of teaching time is on Mathematics and Physics.

The committee examined and recommended on the objectives of the Mathematics courses and their content. Syllabi changes to the respective departments were suggested where appropriate, and a report prepared for the Foundation Year Engineering Committee. There were two points which were consistently made throughout the reported discussions on the content. Firstly, there has been too much necessary repetition of topics. This has resulted in some topics being treated at an introductory level in each of the three courses. This, it was suggested has the effect of both boring the more able students and also in leaving limited time for the study of the more advanced situations. Some of the specific recommendations are included as an appendix.

Unfortunately, Mathematics education is often dominated by the mechanics of manipulation, of the syntax of the mathematical language, instead of the message that the language can convey. "Too often mathematics is presented as a collection of randomly distributed hovels, instead of, as it should be, a magnificent palatial construction" (Bertrand Russell).

In some areas we need to reconsider what terminal objectives are appropriate for the different stages in the students mathematical development. Transfer of the skills from Maths Lecture room to the Engineering Laboratory is clearly not optimised. It may be helpful here to consider the "think concretely" philosophies of the Mathematician and Educator A.N. Whitehead, who postulated that applications should always be demonstrated, and analysis cross referenced to its use as a problem solving tool. Examples here are not difficult to find, as test equipment and computing aids are now available, sometimes as much in PNG as in Western World Institutions.

Conclusions

The objectives of Engineering courses are criteria based, and relate to synthesis and evaluation rather than analytical syntax. Traditional methods of assessment may not be appropriate. In PNG students appear to have developed some undesirable survival techniques which are becoming structural. A use of cross referal into other applied mathematical areas may be useful. A review of the curriculum to include a consideration of these problems would be highly cost effective, since Engineering professional posts in many Departments will be one of the last areas considered for localisation. The recommendations of the Foundation Year Engineering working party should be fully implemented.

Acknowledgements

I thank other staff members for their comments and advice, in particular, Mr. L. Carter, Mr. P. Robinson, Prof. P. Greenwood.

Reference

1) Report of the Foundation Year Engineering Mathematics/Physics Review Working Party, October 1982, PNG University of Technology.

Appendix - Working Party Report to Foundation Year Engineering Committee

Some Recommendations and Conclusions

These are presented in the order that they arose, and not necessarily in order of importance.

That the F.Y.E. Committee communicate to the appropriate syllabus advisory committees of the National High Schools that

(i) there is concern at the understanding of basic mathematical topics of the N.H.S. graduates. There also appears to be too many topics covered too superficially. There is no general purpose for teaching calculus and vectors at that stage, and it would be appropriate to delete these topics and to treat the other topics in more depth. In particular, trigonometry, algebra, indices and graphical techniques are not well understood and the needs of the engineering courses would be better served by a more thorough treatment of these topics.

(ii) there is a very real deficiency in the experimental physics background of the students. A much better introduction to the physics courses at university would be provided by a more extensive range of practical experiments on the basic physical concepts of force, friction, gravity and uniformly accelerated motion.

The broad philosophies which the courses should adopt are as follows

MA139 will teach mathematical techniques and approaches in a generalist and theoretical framework, to provide skills which will be required in the other Foundation Year Engineering subjects.

PH150 will provide the practical and experimental introduction to those physical phenomena prerequisite for engineering studies.

EF110 will link the physical concepts with mathematical models and attempt as many applications as possible.

That the policy whereby the physics topics are taught to different groups at different times be discontinued, and that all groups should cover the content at the most appropriate time in relation to the other subjects.

That physics attempt to reduce the number of sessions on routine mathematical type problems, and increase the emphasis on practical, student-centred experiments, and on problem solving with hypothetical situations and interpreting physical models.

That the EF110 course should ensure that topics are covered after, and in relation to, the appropriate physical and mathematical introduction.

ENGINEERING MATHEMATICS: A PROBLEM OF LOGIC?

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ABSTRACT

The engineering students are an important class of users of mathematics. It may be helpful to indicate some of the areas of weakness in students at Unitech and to examine possible causes and or remedial action for those weaknesses.

Perhaps the best way to highlight the problems is by reference not to mathematics or to engineering but to computing. I believe that the experience of the Civil Engineering Department in the field is shared by other departments.

Suppose we generate a set of "n" numbers within a routine and we wish to compare these sets and form a maximum set such that the r^{th} element of the maximum set is equal to the greatest value of the r^{th} element of all the sets. Such a problem arises in design. We have to design all parts to sustain the worst action at that part. Many students who appear motivated, and who have obtained pass marks in conventional mathematics classes have difficulty in writing the algorithm to achieve this.

Within a double loop all we have to write is

```
IF  MAX VALUE (J) < VALUE (J)  THEN
    MAX VALUE (J) = VALUE (J)
```

Perhaps student weakness results from the absence of practise in "lateral thinking" or to put it differently the concentration on analysis at the expense of synthesis.

This paper urges enquiry into means of increasing awareness of the logical structure of knowledge, and suggests some procedures which should be tested.

Many people are aware of Bloom's taxonomy of educational objectives, and of the hierarchy of skills ranging from recall (level 1) to synthesis (6) and evaluation (7). There would be general agreement that to assimulate knowledge, to be aware of the interactions of knowledge, and to be able to infer behaviour in an area not previously encountered is of much greater value to any country whether developed or developing than the skill to perform repetitive functions. The need for people who know why certain routines are performed as opposed to those who know how to perform those routines, and with it the ability to choose, or even to create new routines is self evident.

The tertiary institutions in Papua New Guinea are inevitably creating the professional elite on which future government and administration must rest. And yet too often the criticism is made that the feeder institution only teaches and tests the skill of recall. This criticism is made by employers of the universities, by the universities of the National High Schools, by the National High Schools of the Provincial High Schools and so on. Perhaps each of these sections should look within their sphere of action; more would result than from complaining about the raw material.

If we examined our own work against Bloom's classification, this would be somewhat subjective, and even arbitrary, but we would all find a high proportions of teaching and testing in the lower levels. It is easy to see why. We are all likely to be judged harshly if we have too many failures. We can reduce the risk if we make tests as predictable as possible. The most predictable test is that closely related to the rote learning which has been tested by worked examples ad nauseam. If we do not resist this pressure we have no cause to criticise other levels of teaching institutions. We should therefore move away from skill level 1 as soon as possible. Even

within a self contained narrow topic we can move to level 2 (translation). For example having taught the conventions for handling $\frac{a}{b} + \frac{c}{d}$, we can move by changing the language and asking for two thirds of 60%. We can also break the habit of rote learning by problems such as "John owns a two thirds share in a boat worth Kx. He wants to fly and has the chance of a one fifth share in a plane costing Ky. How much in cash must he also offer? Or should it be bike and canoe.

At university level we find student difficulties with simple algebra, e.g. $(\frac{1}{R_1} + \frac{1}{R_2})$. No doubt the students have performed this form of algebra often in the safe context of this isolated topic. Recall of this routine under pressure of "multiple choice" tests (Is it $\frac{1}{R_1} + \frac{1}{R_2}$ or $R_1 + R_2$?) is a very different level of difficulty. I do not think that to say "they have forgotten it" is helpful, even if true. Rather we should ask how often we test and reward the skill of assimilating diverse strands of information.

What use could we make of short questions like is 2^8 greater than 8^2? We find students who have difficulties with indices, even to the point of being unwilling to use the prefixes kilo, milli and micro, and students who do not really know why $(\sin x)^2$ is not $\sin (x^2)$. No doubt these students have performed set tasks without undue difficulty, but how much of their earlier testing has required acts of judgement?

Clearly there can be no revolution in secondary and tertiary education. These must produce if possible more than the present number of graduates each year. It is not a question of making tests more difficult in that more candidates fail, but rather that we seek to reward different skills. It is conceivable that the distribution of people within the tests might not be very different. The same people might fill the upper quartile however they were tested, but their expectation of career responsibilities might be better.

I believe that for all grades higher than community schools, rewards should be seen to be given for ability to coordinate knowledge. Clearly coordination cannot come before the more narrowly conceived skills are taught and reasonably well understood. I have never regretted learning the multiplication tables.

I believe that teachers at all levels should analyse their material in terms of Bloom, or in the concrete – transitional – formal classification of Shayer and Adey.

I believe that judgement should be invoked at the earliest possible stage with the greater use perhaps of inequalities, and the use of multiple tests. Suppose a function representing say profit must not fall below one figure and the function representing time for repayment must not exceed another figure. The independent variable or variables could then have a permitted region, and depending on the nature of the functions, substitution in or solution of the equations would test logic as well as algebra.

I accept that teachers work hard with inadequate resources. In these conditions it is easy to follow the safe route of rote learning and recall. To do otherwise is to run risks and to give yourself more demanding work. But the nature of the students in Papua New Guinea, and the rapid rate of development expected of them requires that they should reach Bloom's objectives 6 and 7 as early and as often as possible.

There is one other aspect of mathematics which is perhaps overlooked – the interaction which is required between language (english) and the symbolism of algebra.

When required to give a formal proof of a relationship very few Unitech undergraduates can construct correct sentences. The equal sign is used indiscriminately and the sense that some statements can be seen as logical

consequences of others, justifying the use of words such as "since – then" seems very little developed. The conditional assignment available in BASIC – "If – then – else" seems to give problems which are as much related to language as to logic. Should we at the same time as teaching towards synthesis look also for synthesis between the worlds of language and of symbols?

I have in mind the use of problems written in words and numbers in which the students has to formulate the equation. The enemy of progress is complacency. It is easy to blame the very different cultural background of the students for attitudes to work and learning, but I do not think there are any grounds for complacency within the grounds that Government expects the University to operate. The Civil Engineering department has not yet produced a citizen graduate who has been able to sustain further study and been sufficiently interested in some part of engineering knowledge to want to pursue the topic beyond the undergraduate level. These are the people from whom academic staff are selected in the developed world. This failure is common in the mathematical and applied science fields in Papua New Guinea, so we ought to be more willing to examine the nature of the tasks that we set our students. If we don't inevitably some other body will set up to do the job.

THE MATHEMATICS EDUCATION CENTRE: AN OVERVIEW

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ABSTRACT

The Mathematics Education Centre of The Papua New Guinea University of Technology is an independent research centre with the wide brief of upgrading the learning and teaching of mathematics throughout the country. The work of the Centre can be conveniently divided up into four overlapping areas; curriculum development, acting as a resource centre, training of nationals, and carrying out basic research in the area of mathematics education.

This paper covers each of these areas in turn, and in so doing seeks to identify areas of educational need. Within the area of curriculum development 'hands - on work' for historical reasons is concentrated on post year 10 institutions. Some detailed projects have been undertaken in 'trade' or 'vocational' mathematics, with the other main area of concern being with the National High Schools. The Centre operates as a resource centre for both the university's mathematics department, and to a lesser extent surrounding schools. Two offset printing machines with peripheral devices are operated in the Centre's print room. Text books, teaching and learning aids, a micro computer, and a small library are also provided by the Centre. The training of national personnel is a high priority within the Centre. The country is seeking to move from its multitude of traditional cultures to a technological society. The training of nationals who have some grasp of research techniques in education is seen as important.

In some ways, the research carried out by the Centre forms the basis of much of its other work. It is the one place in Papua New Guinea where long term projects within the area of mathematics education can be mounted without the worry of money running out, a worry often experienced by those working on projects funded by overseas money. The paper concludes by giving a brief summary of research carried out on attitudes and mathematics, language and mathematics, and spatial ability and mathematics.

The Mathematics Education Centre (M.E.C.) at the Papua New Guinea University of Technology (P.N.G.U.T.) started operations at the beginning of 1977. Its precursor was the Mathematics Learning Project (M.L.P.) which developed the mastery learning system used for teaching mathematics in the preliminary and foundation years at the University of Technology (P.N.G.U.T). The M.L.P. personnel designed the first editions of most of the units still used by the Mathematics Department. The project was not exclusively interested in the teaching and learning of mathematics at P.N.G.U.T. It also mounted research projects at school level looking at curriculum materials, cognitive development and spatial ability research.

The M.E.C. has always been an independent Centre of P.N.G.U.T., although working in close cooperation with the Mathematics Department. The main financial supporters of the M.E.C. originally were The British Council and P.N.G.U.T., but since 1980 it has been solely funded by the University.

The work of the Centre can be divided into four, overlapping areas; curriculum development, a centre of resources, training, and basic research. This paper will briefly comment on the Centre's work in each of these areas, and in so doing point out the educational needs associated with each.

CURRICULUM DEVELOPMENT

Specific curriculum development projects have been restricted to post-year 10 institutions. This demarcation arose when the Mathematics Education Unit (M.E.U.) was set up at University of Papua New Guinea in 1980. The

M.E.U. was to have a specific input into the curriculum development of the Community Schools (years 1-6) and Provincial High Schools (years 7-10). As well, the Curriculum Unit of the Ministry of Education has been very active in this area. It is also a natural area of involvement for the Teachers' Colleges. The input into curriculum development at pre year 10 level by the M.E.C. is therefore mainly in a support and an advisory role, rather than a 'hand-on', doing role. The support role is performed mainly through inservice work, whereas the advisory role is based on the basic research work carried out at the Centre. These aspects will be commented on later in this paper.

One of the key areas for the Centre in curriculum development is the National High Schools (years 11-12). The Centre has participated in the revision of their three mathematics syllabi over the last three years. The Director has also been asked to coordinate the setting of the four year 12 papers, and the marking of the examinations at the end of the year. Building up and sharing of written resource material between the schools has also been encouraged. There is now a file in each school containing copies of teaching notes and example sheets from all schools. This is leading to a growing awareness of the emphasis each school wishes to put on various aspects of the syllabi, and hopefully a common interpretation of the syllabi. The file will also serve as a useful resource when text material is produced. Each school is visited at least once per year when there is opportunity to talk about the mathematics programs. Information on appropriate materials and texts is also fed out to the schools when these come to the attention of the Centre. Other rather basic helps are also provided. For example graph paper, printed on the Centre's offset machines, is sold at cost to the schools.

Curriculum development is often narrowly viewed as producing student materials. There obviously needs to be a build up to such an end product. In the late seventies the National High Schools were bordering on teaching different courses in mathematics. The syllabi have been revised, but are still not perfect. Perhaps it is nearly time to start producing text books for the National High Schools' mathematics courses. A 'tuning up' of the syllabi could be undertaken at the same time. There is certainly need for such a book to give stability in the continuing flow of expatriate teachers through the schools, and guidance to the inexperienced national teachers teaching these courses for the first time.

The amount of curriculum development work for the P.N.G.U.T. Mathematics Department has lessened considerably since the days of the Mathematics Learning Project. However, the Centre has been able to respond to requests on methods in writing units, layout of material and the development of aids to facilitate the teaching of mathematics. It is also on the lookout for resources which seem to be relevant to the teaching of mathematics at the University of Technology.

The other role which the Centre seeks to perform at P.N.G.U.T. is to sensitize all university staff to the mathematical background that entry students have. Increasingly the entry students are ex-year 12, not ex-year 10. As well, the courses in both National and Provincial High Schools are in a state of flux. In Papua New Guinea it is vital that the students experience courses which flow on from one to another. Universities cannot afford the luxury of teaching a body of knowledge which meets a mythical criteria of what a first year engineering student, for example, must know. The University courses must be continually attuned to what the students do know when they begin their courses.

The Centre appears to have a growing role in curriculum development beyond mathematics at the University of Technology. It has been able to respond to requests from other departments on educational matters affecting their programs, especially in the area of evaluation and diagnosis. Perhaps in the future the M.E.C. will be able to play a useful expanded role in this area, being the only section or department of the university specifically recruiting personnel for their expertise in education.

The third area in which the Centre has a role in curriculum development is an area mainly outside the mainstrain of the education system and yet vital for the development of the country. It encompasses the vocational training institutions, be they government department training schools like the P & T Training College, Agricultural Training Centre or the Schools of Nursing; private enterprise training schools such as B.C.L.'s or B.P.'s; or the Technical Colleges. The need for help in these institutions arises from the type of instructors employed. They are predominantly people who have good experience in their relevant vocations. Although these instructors are able to judge the capabilities of trainees when placed in the work situation, they often do not have the skills to develop a training course, especially when it comes to the more academic aspects of the training like mathematics. Hence these instructors need help.

Confronted with this scenario, the M.E.C. has developed a rationale which concentrates on the specific task of training, rather than the more global, educational aims appropriate to schools. The two main principles are to isolate and teach only that mathematics which has particular

relevance for the vocation, and secondly to draw introductory and practice examples as much as possible from the vocation's work situations. We do not believe that some generalist course in basic mathematical skills (whatever they are), which attempts to reteach the mathematics courses from years 4 to 10 is appropriate.

To this end two projects which outline the approach taken in different circumstances will be noted. The first is the 'Mathematics for Student Nurses Project'. In the eighteen Schools of Nursing, the mathematics course is often taught by outsiders employed for the first few weeks of the first year course. This has led to a wide variety of emphases being placed on different aspects of the syllabi. The M.E.C. was approached by the Health Department and requested to help in upgrading their mathematics courses. It appeared at that time that there was no one available in the Health Department who had the capabilities for writing appropriate material for their schools. Hence the M.E.C. developed over a period of three years a text book for the Schools of Nursing, written specifically for that vocation.

In most of the other government departments, or in private enterprise, there were fewer institutions teaching the same course. As well, there seemed to be some expertise available. Hence for the last three years the M.E.C. has mounted a 'Writers Conference' which gives guidance to instructors in writing their own course material for the particular vocations in which their institutions train students. This 'conference' is really three rolled into one; a two-day preliminary workshop at the beginning of the year, a week-long live-in conference at mid-year where participants actually write for the whole time, and a follow-up day seminar later in the year. This year (1983) the M.E.C. has received a grant from UNESCO to continue this work. The whole area of

developing specific, useful curricula for future skilled workers is a vital need. The M.E.C. is only scratching the surface.

The above is not an exhaustive treatment of the M.E.C.'s work in curriculum development, but does sketch in the main lines. Much other incidental work goes on at the individual level of working with particular teachers, or giving one off in-service lectures or workshops.

CENTRE FOR RESOURCES

Another role of the M.E.C. is that of a resource centre. The Mathematics Department's courses were once strict mastery learning programs. Although individual courses have undergone many changes, the programs still involve a lot of printed material. The M.E.C. runs two off-set machines, collator, etc. Approximately 60 percent of their time is taken up with the mathematics course materials. As well, the M.E.C. acts as the distribution centre for this material. One of the great advantages of this system is the potential for continual revision of the modules and units as better ways of presentation are evolved, and as the student population changes. With the close association of the Centre and Department, this potential can be brought to fruition.

The M.E.C. also runs a Resource Centre which among other things houses the aids and equipment which are required by the university mathematics courses. Some units of work require students from time to time to visit the Resource Centre and work with learning aids.

The Centre also houses other equipment and a fairly extensive range of text material which is available for university staff to consult, and indeed for teachers in surrounding

schools. It also is the basis for many of the workshops and inservice sessions run by the Centre. For example, recently two sessions were organized for the Community School Inspectors from the Morobe Province. The Morobe Mathematics Panel, the local mathematics teacher's group, is also a frequent user of the Centre.

Even with the recent production of text material for years 7 and 8 in mathematics, and the future production of material for years 4, 5, 6, 9 and 10, teachers and schools must become aware of the whole host of resources available for teaching mathematics. Perhaps one of the real needs is to build up a stock of relevant and useful texts in each school, or at central locations, which teachers can use in their own courses. At least this resource should be available for the inspectors and curriculum officers in the provinces. Perhaps the fundamental problem is the lack of awareness of just what is available. Teachers cannot choose in a professional manner the methods they will use to teach a topic if they do not realize they have a choice. The M.E.C. Resource Centre is a small beginning in building up this awareness.

The Centre has also developed some computer facilities with an Apple micro computer and, in combination with the Department, terminals to the main university computers. Although used mainly for research purposes at present, with this equipment there is the potential of developing materials for schools once a rational policy of computer usage is worked out for them. With each National High School now in procession of a micro, and a number of Provincial High Schools also having one, such a policy can no longer be delayed.

TRAINING

Of the eight and a half positions in the M.E.C., seven are occupied by national staff, three of which work full time on the projects of the Centre. The three are qualified teachers, and have had varying degrees of experience in the classroom. All have now had experience of collecting and collating data, of working on the computer, of running workshops and other inservice sessions. Various upgradings of their academic qualifications are in progress. I mention this to highlight the belief that a small but significant number of people within the education establishment should be trained in a broader sphere than simply as teachers, or as theoretical academics. If an administration is to interpret research results within a proper perspective, then there needs to be on hand people who have been out there doing the needed grass root research, in the classrooms and lecture halls of Papua New Guinea, and incidentally gaining a very different view of the education institutions than is available from the desk of an administrator, or as a teacher or lecturer at the front of a classroom or lecture room. The M.E.C. is one of the few places in the country where that sort of training can go on, and perhaps the only place which offers stable, long-term positions which are not just created for a particular project funded by overseas money.

Another area of training in which the Centre is involved is inservice work with teachers. Although this is an area where extensive work needs to be carried out, it is not at the top of the priority list for the M.E.C. The Centre has not the resources to mount sustained, long term training programs. This area was of course to be taken over by the M.E.U. when it was set up. Hence at that time the M.E.C. started to concentrate far more on other areas of need. The demise of the M.E.U. was a great loss to the in-service work throughout the country.

Since then, the running in this area has been left to the Teachers' Colleges and the Curriculum Unit of the Ministry of Education. However the M.E.C. does support extensively the work of the Morobe Mathematics Panel. As well, in the last three years, M.E.C. staff have mounted nearly thirty percent of the sessions run at the yearly National Workshop for Mathematics Teachers. Centre personnel from time to time run one off sessions for various schools or Teacher Centres when requested, and they are available to run cooperative projects with other groups.

The priority area of M.E.C. inservice work is more for key people in the system; that is, the curriculum officers, inspectors and for Teachers' College personnel. Seminars and workshops are run for such groups from time to time. A recent development, working with the Morobe Community School inspectors, has already been noted. A critical need is support for lecturers in a number of the Community School Teachers' Colleges. Although the M.E.C. would like to become more heavily involved in this area, there is a real danger of over-extending our meagre resources, and as a consequence not doing the job properly.

RESEARCH

So far this paper has pointed out a number of the needs in the area of mathematics education. These have ranged from developing text material for the National High Schools and vocational training institutions, to an adequate and comprehensive in-service training scheme for teachers. The needs noted do not by any means define all problem areas. The satisfactory completion of texts for years 9 and 10 may well prove difficult for instance. These needs are only those to which the M.E.C. has been able to devote some time.

In attempting to find solutions, appropriate research must be carried out. Part of the meaning of 'appropriate' will be research carried out in Papua New Guinea. Too many supposed solutions have been proposed by visiting experts who have not had the time, or sometimes the will, to get out into the schools and become acquainted with the situation there. Dienes' input would be an example of this. Some strategies which work in England or Australia may well be inappropriate in Papua New Guinea because of language problems, different attitudes of pupils and teachers, difficulties in mounting in-service training, or an inability to supply schools with the ongoing, usable materials they would need to make an aids based program function.

The fourth role of the M.E.C., and in some ways the most crucial role, is to attempt basic research projects which will hopefully go some way to the finding of long term solutions to these needs. There is an unavoidable conflict of priorities in the system at present. Clearly at all levels there is a need for materials and mechanisms which will give stability to the syllabi that are taught by a teaching force which is comprised of young inexperienced nationals, and/or highly mobile expatriates. They want to know what is expected of them; at least what content is to be taught, and for the inexperienced teacher either per se, or in Papua New Guinea, what methods are most appropriate.

By comparison, in Victoria, Australia, there are no prescribed syllabi in the schools at any level, except at year 12, and even at that level there are options available. However, there is a long standing tradition that certain topics are taught at certain levels and most schools, more or less, follow the tradition. This is reinforced by the fact that the majority of texts available are written with that understanding. Also

the teaching force is older, more settled and perhaps likes doing what it did last year and the year before that. In Papua New Guinea there is no such tradition, and syllabi have been regularly changed. But such changes should be based on adequate research of the learning styles and teaching styles which are appropriate for Papua New Guinea. But, to complete the vicious circle, there is not time for such research to be completed. Teachers need guidance and materials to use in their mathematics teaching now.

Such projects as the Indigenous Mathematics Project have provided useful data on basic questions to do with the teaching and learning of mathematics in Community Schools. But clearly much more research is needed. It is also perhaps no surprise to find that some of the M.E.C.'s major research is directed at these early years of schooling. To a certain extent, work at the top end of the system is 'band-aid stuff', although very important for the present population of students already in the system. However in the long term, ten years or more, the problems must be sorted out at the Community School level. It is there that the fundamental problems lie, and they must be tackled. Hence the M.E.C. does not apologize for devoting part of its resources to research at the lower end of the system.

The unique role then which the M.E.C. seeks to play is an ongoing thrust in this area of basic research, without the worries of the money running out on such and such a date, which is ever present with overseas funded projects.

Results of such research need to be disseminated to the people who count. Thus the M.E.C. has two series of reports; the Mathematics Education Centre Reports aimed at a wide general audience, and the Mathematics Education Centre Technical Reports which are sent out to specific

groups. Probably, however, the more meaningful reporting of results occurs by word of mouth when qualifications and implications can be spelt out.

To this end the M.E.C. has hosted a conference on mathematics education for the last three years, not only to report on our own research, but to give opportunity for comment and discussion on other mathematics educational matters pertinent to the fairly broad group which has gathered each time. Of course one conference per year is not enough. However there are other opportunities to publicize the results of our research during the inservice work carried out by the M.E.C., with comments made during the National High School, Provincial High School and Community School Syllabus Advisory Committees' meetings, and so on. Perhaps the most important method remains the personal contact with key workers in the Curriculum Unit of the Ministry of Education, at the University of Papua New Guinea, the Teachers' Colleges, and curriculum advisors and inspectors at the provincial level. For information on research to flow on quickly to the curriculum developers and teacher trainers, such contacts are vital.

It is also necessary that in reporting on research, the implications for curriculum development and teacher training be drawn out. It should not be the common procedure to leave a report on research at an academic level in Papua New Guinea. The arguable point is how far should the implications be drawn out by a researcher who is not part of the system, and who may not fully appreciate the intricacies of the system in which the development must take place. On the other hand, comments by outsiders, for example, M.E.C. personnel, may well bring a new useful perspective to bear. We hope it does. Our main rule of thumb in this is to draw out fairly broad implications and leave the detail for face-to-face discussion.

There is not room in this paper to give full details of the various projects in progress at the Centre. Rather sketches of some of them will be given to outline the main interests of the M.E.C. Fuller details are given in the various publications of the Centre.

An area of growing interest in the M.E.C. is that of attitudes and mathematics. One of the main differences constantly highlighted by lecturers and teachers from western countries is the way a student's personal attitude to a lecturer or teacher will have on the amount of work the student will do. Put crudely, if students do not like you, then they will not work so as to punish you. In other words, they will not buy what you are selling. It has been conjectured for some time that such attitudes of Papua New Guinean students to mathematics, and study in general, is somewhat different to, and perhaps a more important influence on their learning, than with their western counterparts.

An early study of the attitudes of first year university students to mathematics indicated that they had a positive attitude to the subject, but this was not very pronounced. More disturbing was an indication that these students viewed mathematics as only knowing and applying rules.

In another study based on the literature of 'attribution theory', Papua New Guinean students' reasons for their successes and failures in mathematical situations were investigated. In studies of western students it has been shown that about 85 percent of reasons that students give for successes and failures can be classified into four categories; task difficulty, how much effort was used, innate ability, and luck or environmental circumstances. It has also been shown that from about year 6 onwards, boys and girls develop quite different patterns of reasons

for success or failure. It seems that for Papua New Guinean students there is no difference in the reasons given by boys and girls up to grade 9. As well, although the reasons for success follow the western pattern, for failure situations, they do not.

A further study carried out at the Centre sought to explore whether Papua New Guinean university students perceive 'good' lecturers to have different characteristics than their western counterparts. This was done by using survey instruments prepared for American and Australian students, and asking the Papua New Guinean students to fill them in. The analysis of the results showed there was a marked degree of overlap between the two student groups. However there were important differences. For example, the Papua New Guinean students selected items related to the 'lecturer as an individual person' rather than items related to a lecturer's 'organizational capacity' as western students had done, when asked to identify five items from a list of 63 which most charaterized a lecturer. In comparing the results of the factor analysis of Papua New Guinean responses with that of the western students' responses, the most noticable difference was that in the Papua New Guinean analysis, items related to the lecturer as an individual and items specifying organizational and planning capacity were linked, whereas the western students quite clearly differentiated between these facets of a lecturer's presentation. In the light of these results, it was suggested that expatriate lecturers, the majority of the university's teaching force, would need to acknowledge that their students do see the world in a different light, even to the extent of perceiving lectures differently to their western counterparts. Second, staff members may need to allow themselves to be seen as people, rather than cold academics. Finally it may help if staff talked about what they saw lectures as being and then

their own role in them, so that the students might appreciate that there is a different perception available. In this way the lecturer and his students might well enter a mutual process of learning. This last point is probably the most difficult of all. Papua New Guinean students are often reticent in the presence of teaching staff. Therefore the impetus for such an interaction must come from the staff.

It is hoped that the results from the above type of research will give lecturers, teachers and curriculum developers a deeper understanding of what Papua New Guinean students are like. It may be that with the realization that national students are different to their western counterparts, that research into teaching methods appropriate to Papua New Guinea will be undertaken.

A second area of research interest for the M.E.C is that of error analysis. As the term implies, this is an area of study which seeks to establish the specific types of errors made by students, and then to suggest teaching methods to overcome them. One particular stream of error analysis research in mathematics education has focussed on written problems, and thus has involved a study in part of the interrelation of language and mathematics. This approach in a country which has seven hundred and twenty indigenous languages, plus a few more lingua franca, plus an official language, would on the surface appear to be a useful tool.

In a number of pilot studies so far carried out it seems that students are having real problems with the simple reading of written mathematics and comprehending what they read. This is before they even try to process the mathematics required by the written problem. Such results support and perhaps generalize the findings of

earlier work carried out in M.E.C. The earlier work concentrated on the specific area of understanding the ideas of 'more and less'. It concluded that the development of these ideas followed the same pattern as that in western countries, but development time was prolonged by something like four years. Probably the next major project of the M.E.C. will be a thorough investigation of the problems year 6 students have with written mathematics using the techniques of error analysis.

A third area of interest in the M.E.C. is visuality and spatial ability. A longtitudinal project started in 1981 and due to run until 1983 has as its aim the investigation of the preferences National High School and first year University of Technology students have for processing mathematical information. This grew out of a 1980 M.E.C. project which suggested that those engineering students who had a preference for an analytical approach to mathematics scored higher results in mathematics than those with a more visual preference. A further analysis of the data seemed to indicate that entry, post year 12 students out performed entry, post year 10 students because of their preference for analytic thinking. It was conjectured therefore, that successful students may well change their preferences for processing mathematical information, becoming more analytical with time. This study is an attempt to follow through this conjecture.

Data was collected from year 11 and 12 major mathematics students in 1981, and from all year 12 and first year university students in 1982. In year 11 all students complete a compulsory mathematics course. In year 12 students are free to choose a major course of eight hours per week, or a minor course of four hours per week.

The 1981 data has been analysed, as has the year 12, 1982 data. The 1981 data suggested that there was no difference between the year 11 and 12 groups as far as preference for processing mathematical information was concerned. Secondly, although the variable for measuring a preference for visual or analytic processing was a poor predictor for the end of year 11 mathematics results, in year 12 this variable became the best predictor. However the variance explained was still quite small. The other variables entered into the regression equations were measures of short term memory space, final year 10 mathematics and english results, a test of cognitive level based on the Australian Council for Education Research's 'Operation Test', and a set of written mathematical problems.

With the 1982 data, it was possible to look for the changes occurring in individual students by comparing their year 12 results with those obtained in 1981 when these students were in year 11. Although a final report has not been written on this data as yet, at this stage it seems that the students who took major mathematics at year 12 level did not change their preference for processing mathematical information. But the minor mathematics year 12 students did. They changed to a more visual preference. The results from regression equations imply that the analytic/visual preference variable was a poor predictor of performance for both the major and minor groups of year 12 students.

What then can be made of these results? One of the difficulties in interpreting such data is using the end of year score as the dependent variable in regression equations. Obviously the external examination papers will change in emphasis and perhaps a little in style, thus not giving a truly repeatable situation. The

advantage is that the results are able to be related directly to the situation that the students and teachers find themselves in. It may be this fact of the external papers changing a little which explains the analytic/visual variable being the best predictor for the 1981 year 12 major students, but being an insignificant variable for their 1982 counterparts. One of the more important comments about the 1982 major paper was that it was a little more straight forward, with perhaps more emphasis on the mechanics of mathematics rather than problem solving. The drop in importance of this variable would follow logically from the type of variable which it is. However this is a very tentative suggestion only: a suggestion which does imply a possible future avenue of investigation.

The data seems quite clear, that there was not a move along the scale by successful mathematics students to a preference for analytic thinking. Successful students in this context are those who opt for, or more likely are chosen to do, major mathematics. Thus the original conjecture based on the 1980 studies does not seem to be confirmed. Rather those who were not succeeding may well have been relying more and more on visual thinking. Perhaps the style of teaching in year 11 and 12 emphasizes analytic thinking to such an extent that those who can not cope in fact reject it and revert to a style of thinking which was more natural for them. Such a conjecture clearly needs a study of its own. If such was shown to be the case, the implications for teaching mathematics would be far reaching.

Lack of space prevents the relating of other research projects either by individual M.E.C. staff, or in cooperation with others. Some work has been carried out on traditional ways of thinking about 'time'. Conjectures have been put forward in relation to curriculum development in a developing country; that detailed syllabi, set texts and a specific examination system are vital in an education system which lacks a definite framework given by a stable, experienced and qualified teaching force as found in developed countries. Research work has also grown out of curriculum development as well. For example within the 'Mathematics for Student Nurses Project' some work has been directed at whether scores on written tests of calculations correlate with tests in practical situations. It has been found that the scores are positively correlated, although the types of errors made in the practical situation are not predictable from the written tests. Further details of this research can be found in the Centre's publications.

CONCLUSION

The M.E.C. has been seeking to identify needed areas of research and/or curriculum development. With such a large array of problems, choices must be made. In making these choices attention must be given to whether the Centre has adequate resources to handle a particular project. We are relatively small with the bulk of members still being trained, and a fair commitment in time to being a resource for the University of Technology Mathematics Department. There is always a danger when the problems are large and entrenched of attempting the impossible. All the M.E.C. can do is provide relevant data, give advice, and perhaps provide resources. In some special areas we may go on and be directly involved with the curriculum development. However this last stage

is normally better performed by people in the relevant system.

What then is a way forward? Well there is no slackening in the need for research. The more that is unearthed, the more tangled becomes the strategy of finding adequate, long term solutions to the problem of improving the learning and teaching of mathematics. In a country such as Papua New Guinea where there is much pressure for development, and hence for upgrading of courses and the provision of the consequent materials, there is the danger of the 'oversea solutions' being applied with scant attention paid to the local conditions. Papua New Guinea is very different to England and Australia. It is also very different to African and Carribean countries. This is not always clearly recognized. Indigenous based research must go on to find the solutions for Papua New Guinea.

Finally, there is a continuing need for the researchers, the teacher trainers, and the curriculum developers at all levels of the system, even through to those who develop the university courses, to keep listening and talking to each other, and consciously inviting the opinions of the others. If we stop talking and listening, we will be in a big trouble.

A STUDY OF THE ABILITIES OF SENIOR SECONDARY STUDENTS AT PROPORTIONALITY TASKS

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ABSTRACT: The ability to handle tasks requiring proportional reasoning is an important prerequisite to vocational, technical or higher studies in mathematics. This paper reports a study which indicates that grade 10 leavers experience very real difficulties with proportional reasoning tasks. The implications for post-secondary courses are discussed.

INTRODUCTION

Very many of the practical applications of mathematics require the use of proportional reasoning. Often secondary courses give scant attention to the teaching of ratio and proportion. Perhaps teachers do not appreciate the significance of proportionality in practical situations. Some of the mathematical applications which require the use of proportional reasoning are as follows:

> costing; wage calculations; weight/price comparisons; currency conversions; measurement unit conversions; scale reading; map reading; scale drawing; scale model construction; surveying; mixing quantities according to a fixed rate such as in recipes, concrete, fertiliser ; chemical solutions; balancing chemical equations; physical situations such as Hooke's Law, Boyle's Law, Ohm's Law; musical scales; gears; speed; pendulums ; pulleys and levers; models where growth rate is dependent on size; etc.

It is almost difficult to construct a real problem situation where proportional reasoning is not required. Certainly there is no vocational or technical course which could ignore the need for the students to perform tasks requiring proportional thinking.

This paper reports a study of the ability of grade 10 and post-grade 10 students at proportionality tasks and discusses some of the implications for mathematics courses.

STUDIES OF RATIO AND PROPORTION

Inhelder and Piaget (1958) found that an inability to handle proportionality schema is symptomatic of an inability to act logically in a formal operational mode. Lovell and Butterworth (1966) reported a series of studies of proportionality with early teenage children, in the U.K., of the upper ability range. Their

results indicate the slow development of a concept of proportionality in secondary school pupils. Karplus and Peterson (1970) tested a sample of students, in the United States, from a range of ages from primary to upper secondary. They used the "Mr. Tall" test where two men are measured using one standard, and then the questions relate to the comparisons using a different standard. They concluded that successful proportional reasoning is not achieved earlier than the last years of high school. Shyers and Cox (1978) reported the results of a program to improve the proportionality concepts of American college students. They concluded that very large numbers of college students find proportional thinking impossible. J. Jones (1973) conducted a thorough study of a wide range of proportionality tasks with preliminary year students at the University of Papua New Guinea. He expressed grave concern at the inability of the students to handle proportional reasoning tasks. P. Jones (1980) used the "Mr. Tall" test with first year engineering students at the Papua New Guinea University of Technology. He found that only 30% of the students could perform tasks involving proportionality without further assistance. He showed, further, that their proportional reasoning ability is roughly comparable to middle secondary level students in the United Kingdom.

J. Jones (1973) suggested that the inability of the students to handle proportionality tasks was severely hampering their studies, and that the teaching programs at the University of Papua New Guinea failed to recognise the difficulties that the students were experiencing. Sayre and Ball (1975) conducted a piagetian style study with secondary level American students. They found that

only the very best students were able to operate at a level requiring formal thinking. They strongly recommended that courses be structured so that they do not require formal thinking. P. Jones (1980) went further in suggesting that where tasks requiring proportional reasoning cannot be avoided, it may be necessary to teach some students an algorithm which bypasses the need for proportional thinking.

Inhelder and Piaget (1958) suggested that proportional reasoning abilities develop through clearly defined stages. Karplus et al. (1974) found little evidence of these stages of development, and concluded that the type of response given was more dependent on school experience. Noelting (1980) conducted a study with Canadian students of average ability over a range of ages. He found some evidence of stages of development, but he also commented on development within stages. He suggested that age was less important than other factors in predicting success on a task and suggested that the "complexity of content" is the main variable. Karplus et al. (1980,a) found that performance on proportionality tasks was often related to the structure of the problem; that is, the language and representations used, the format, and the framing of the question. They indicated that it is probably more important to consider task related obstacles than developmental theories. Muller (1978) in a study with young children, in the U.K., also found that the type of question was a significant factor. He concluded that as the complexity of the initial analysis increases so does the difficulty of the proportional judgement.

Noelting (1980) suggested that the numbers used in the questions were a major factor affecting performance. Karplus et al. (1980,b) reported that secondary level students find integral ratios easiest to handle, that equal ratios are easier than unequal ratios and that students avoid the use of fractions wherever possible. Hart (1980,a) reported the results of a large study of the ability of secondary students in Britain over a range of science and mathematics tasks. She found that the level of performance on ratio tasks was surprisingly low. In discussion of the effect of the numbers used on the degree of difficulty of the question, she said that:

> the introduction of non-whole numbers into a problem does not make the question a little harder, but a lot harder (p.101).

She also found (Hart, 1980,b) that children tend to avoid fractions and use addition and building up methods with problems in ratio and proportion. This is supported by Karplus et al. (1980,a) who stated:

> convenience in dealing with numbers which avoid fractions is more compelling than maintaining an appropriate mathematical relationship (p.108).

There are three main conclusions which emerge from these studies. Firstly, upper secondary students both overseas and in PNG find proportionality difficult and programs should be designed to take account of this. Secondly, the complexity of the problems and the way the tasks are structured has a significant effect on the performance of students at ratio tasks. Thirdly, the type of numbers used in a particular question is critical in determining the type of strategies adopted.

These findings indicated directions for this study of the abilities of the grade 10 leavers at these tasks. Firstly the J. Jones (1973) and P. Jones (1980) studies suggest that students at this level would experience difficulties with proportionality tasks. Secondly, if the way the problems are posed produces variations in student responses, it would be desirable to reduce or control this factor. By using a consistent and representative question structure, it was hoped to investigate the influence the type of numbers used. Thirdly, if the type of numbers used consistently affects the degree of difficulty of the questions, it may be possible to draw some inferences about the abilities of the students to handle proportionality tasks. This, in turn, could have direct implications when devising teaching strategies.

THE INSTRUMENT

In a study which aimed to investigate the effect of numbers on student performance, Karplus et al. (1980,a) used the following four categories for the types of question:

(a) Integral ratios within and between the quantities
(b) Integral ratios within the quantities only
(c) Integral ratios between the quantities only
(d) No integral ratios.

However Hart (1980, a) reported that students found certain integral ratios easier than others, and that they sometimes used strategies which simplified non-integral ratios. One observed example of this was a process which she called "successive halving". In the case of a question such as:

6 is to 9

20 is to ?

the student would say "half of six added to 6 gives 9, so half of 20 added to 20 gives 30". Therefore it seemed appropriate to split the category (d) above into two: easy non-integral ratios and hard non-integral ratios. This meant that there were three ratio types being investigated; integral ratios, easy non-integral ratios and hard non-integral ratios.

There were three pilot studies done. Initially the items which were constructed were much too hard. It became apparent during these pilot studies, that students could not complete many questions in an hour. At first it was found, after student interviews, that the items which contained wordy preambles were seriously misinterpreted by the students. On replacing these questions with diagrammatic representations, it was found that these introduced extraneous information and so distracted the students from the tasks. It was also found that changes in question types produced very large variations in student performance.

The final instrument was constructed with a minimum of written information, no diagrams, a limited number of questions and all items relating to the same information. This provided a simple and consistent format. The students were given the following information in the form of a recipe:

To make 4 wholemeal cakes you need -

200 gram wholemeal flour

3 tablespoons sugar

40 gram butter

15 mL water.

Table 1 shows the questions asked, the categorisation of each question and the actual numerical comparisons required.

QUESTION	CATEGORY	RATIOS USED
1. To make 20 wholemeal cakes, how much butter do you need?	Integral ratios within and between	4 --> 40 20 --> ?
2. To make 6 wholemeal cakes, how much flour do you need?	Integral ratios within, easy non-integral ratios between	4 --> 200 6 --> ?
3. To make 5 wholemeal cakes, how much butter do you need?	Integral ratios within, hard non-integral ratios between	4 --> 40 5 --> ?
4. To make 20 wholemeal cakes, how much water do you need?	Hard non-integral ratios within, easy integral ratios between	4 --> 15 20 --> ?
5. To make 6 wholemeal cakes, how many tablespoons of sugar do you need?	Hard non-integral ratios within, easy non-integral ratios between	4 --> 3 6 --> ?
6. To make 5 whole - meal cakes, how many tablespoons of sugar do you need?	Hard non-integral ratios within and between	4 --> 3 5 --> ?

Table 1: Categories of Questions on Proportionality Test

THE AIMS OF THE STUDY

The following questions were considered:

(a) What are the abilities of the students at these ratio tasks?

(b) What is the order of difficulty of the particular question types?

(c) Are there certain non-integral ratios which are easier than others?

(d) Are there students who use correct strategies for the easier questions but use different incorrect strategies for harder questions?

(e) What are the implications for teaching programs at this level?

It was also important to consider whether success at a harder item predicts success at the easier items. If so, there would be two implications for this study. Firstly, that the easier items test prerequisite skills for the harder items. Secondly, the incorrect responses from an individual would indicate the limits of competence of that student on these proportionality tasks.

THE SAMPLE

The test was administered to students in the Schools of Nursing at Port Moresby, Vunapope and Wewak. It was given during class time by their mathematics tutors. There were fifty nursing students tested altogether. The test was also given to 136 students from four grade 10 classes in provincial high schools. All of the grade 10 students from one of the high schools in Lae were tested,

along with two classes at other schools where the students had been randomly grouped. They were tested after they had studied the section of their syllabus on ratio, in the final few weeks of the school year.

THE RESULTS

Table 2 presents the facilities (% correct) for each item for the provincial high school group and the school of nursing group separately.

QUESTION	TYPE	GRADE 10 n = 136	SON n = 50
1.	Both ratios integral	72	76
2.	Integer within, easy non-integer between	74	70
3.	Integer within, hard non-integer between	63	68
4.	Hard non-integer within, integer between	54	62
5.	Hard non-integer within, easy non-integer between	38	50
6.	Both hard non-integer	29	32

Table 2: Facilities for Proportionality Tasks

The results for the two groups appear to follow the same pattern. One quarter of the students were unable to complete the easiest of the questions, and two thirds were unable to answer the hardest type. The facilities decrease in the same way as predicted by Karplus et al. (1980,a) and Hart (1980,b), although there appears to be only a small change in difficulty if there is an integral ratio in at least one part. Considering that these questions had

been carefully trialled to avoid hidden difficulties, the results indicate that the students at this level are not all proficient at even the easiest tasks involving proportionality. Also, there is a very marked increase in the difficulty of the tasks if the ratios used are not integral, which is the case in most practical applications.

The above table gives some indication of the order of difficulty of the items on the test. It is important for the purposes of this study to ascertain whether the items are measuring similar skills. Table 3 presents the value of the biserial correlation co-efficients between each pair of items. This gives a measure of the degree of association between the scores in each case. The results of both the grade 10 and nursing students are combined to provide a clearer indication of the relationship between the question types.

QUESTION	2.	3.	4.	5.	6.
1.	.44	.49	.56	.43	.34(.87)
2.	x	.54	.23(.36)	.27(.59)	.22(.60)
3.		x	.52	.52	.39(.81)
4.			x	.46	.36(.63)
5.				x	.77

Table 3: The Biserial Correlation Coefficients between Questions on the Proportionality Test
(Bracketed Figures are Loevinger Coefficients)

All of the values are statistically significant at the 0.01 level. This means that the indicated relationship would occur by chance with non correlated data only 1 time in 100. Some of the values

are quite high and strongly suggest that those items are indeed measuring similar skills. Certain of the values are lower. This may be due to the use of the biserial correlation which is not as accurate when comparing items of markedly differing facilities. Hart (1980,c) used the Loevinger co-efficient to measure the degree of association between items where there were large differences in facilities. The Loevinger values for the pairs with low biserial correlations are included in table 3 in brackets. These figures are also high and support the suggestion that each of the questions are measuring related skills. (The item 2-item 4 figure is still noticeably lower; there is no ready explanation for this, but it is still statistically significant.)

The above suggests that the items are measuring similar skills. Another important consideration is the extent to which success at a harder item predicts success at an easier item. If these questions form a hierarchy, then it would be expected that a student who answered a harder question would successfully complete the easier items. Table 4 presents the percentage of students who, having got a particular harder item correct, also successfully completed the easier items. The figures in the table represent the percentage who, having got the row question number right, also answered the column item correctly.

ROW QUESTION	COLUMN QUESTION 1.	2.	3.	4.	5.
2.	90	x			
3.	89	92	x		
4.	92	83	73	x	
5.	97	89	89	84	x
6.	96	90	93	83	96

Table 4: Percentages of Students answering Column Item Conditional on answering Row Item

In most cases these figures are high. The number of students who got both 4 and 3 correct was lower. It is not clear why this should be so. It may be , as Freudenthal (1978) suggested, that within and between ratios are more different than is apparent. This could be further investigated, but it does not detract from the implications of these results. Overall, the results appear to indicate that success at a harder item is a very good predictor of success at an easier item, suggesting that the items are hierarchial.

As the questions were asked in the same way, using the same information to control the effect of the structure of the question, it appears that the items are testing similar skills and the students who master the harder items will usually also master the easier items. This suggests that the questions structured in this way discriminate between the students of differing abilities at ratio tasks. It is therefore possible to infer which types of tasks the students at this level might be able to master, and which tasks they would probably not be able to perform. Even

where simple integral ratios are used there are significant numbers of students who cannot correctly answer the proportionality questions. Where non-integral ratios are used, it appears that many students abandon proportional reasoning and try to arrive at an answer by some other method.

STUDENT METHODS

An examination of the methods used by the students reinforces this conclusion. Booth (1980), in discussing aspects of the study reported in Hart (1980,a), found that on easier problems students use low level but correct strategies. For harder problems, many students abandon these methods once the arithmetic becomes too hard. An analysis of the students' worksheets revealed that this is also true in this study. Even though all students had been taught a formal algebraic method for solving proportionality problems (a/b = c/d), only one student used this technique. Over half of the students established a unit factor which could be used as a multiplier, while one quarter of the students "built-up" the answer by repeated addition. Table 5 shows examples of these two commonly used methods.

Question: If 4 cakes require 40 grams of butter, 20 cakes will need how much?

Unit Factor Method	Building-up Method
$4 \overline{)40}$ → 10 20 × 10 = 200	4 - 40 8 - 80 12 - 120 16 - 160 20 - 200

Table 5: Examples of the Common Methods used in Solving Proportion Problems

Virtually all of the correct answers to questions 1, 2, 3 and 4 were reached by one or other of these child developed methods. Those students who also correctly answered questions 5 and 6 continued to use their particular method and were able to manipulate the necessary arithmetic. A number of students, who had used a correct strategy with the easier questions, abandoned this strategy on the harder items and resorted to an incorrect additive or multiplicative method. The incorrect additive strategy was used in this way. For the question, "If 4 cakes need 3 tablespoons of sugar, then 5 cakes need how much?", twenty-five percent of all responses gave the answer as 4 tablespoons (adding 1 to 3). The students choosing an incorrect multiplicative method seemed to pick any two numbers and multiply them. It would appear that the previous methods which the students had been taught were not used to solve these tasks, and the students used intuitive methods to find an answer. Further when they could not perform the necessary arithmetic calculation, many students abandoned the whole concept of proportionality altogether and merely added. If the students abandon the essential aspects of the proportionality

task when the arithmetic becomes too difficult, then it may not be wise to rely on their use of proportional reasoning to solve a range of practical problems.

SUMMARY OF RESULTS

If the effect of question structure is controlled, over one quarter of the students at this level are unable to perform even the simplest ratio calculation. Only one third are able to complete the tasks when hard non-integral ratios are used. Integral ratios are easier than non-integral ratios. Non integral ratios where halving and doubling can be used are easier than other non integral ratios. Little difference was observed whether the ratio is within or between the quantities. Many of the students who correctly answered the easier questions abandoned their successful strategy and tried unsuccessfully to solve the harder items by some other method. It appears that the items were measuring the same skills and that the results do provide a reasonable indication of the types of proportionality tasks which students at this level are able to perform. Overall, it appears that the grade 10 leavers are far from ready to master proportionality tasks which will arise in the work situation.

IMPLICATIONS FOR POST-SECONDARY MATHEMATICS PROGRAMS

The ability of the students to handle tasks involving ratio and proportion is a critical factor in the design of a program to teach technical or vocational mathematics. From the above results it appears that if the question format and initial information are

controlled, then the numbers used have a major effect on the way students tackle tasks involving proportionality. One quarter of the subjects experienced difficulty with the easiest tasks, and where non-integral ratios which cannot be easily decomposed were used, two thirds of the students were unable to respond successfully. This finding is consistent with other studies of these skills with students at equivalent levels. The majority of the students did not use traditional methods for solving these problems but used naive or intuitive strategies, which were successful only when the arithmetic was simple.

It would appear from the literature that the students' ability at proportionality tasks is dependent on their experiences. It would seem reasonable to suggest that programs seeking to develop students' proportional reasoning skills should provide a range of activities and exercises designed to stimulate awareness of the general concept. These could include both examples and counter examples. Practical experiments involving ratio would also appear to be important. Unfortunately, there are at least two factors which indicate that the post-secondary courses may be unable to provide these necessary experiences. Firstly, the available teaching time is very limited. There is usually insufficient time to treat all of the direct applications of mathematics, without embarking on a program to develop more general, formal concepts. Secondly, the available staff in the colleges may not be willing or able to use such an approach. Many lecturers have substantial trade experience, but little teacher training and have usually not completed any mathematics studies in addition to their pre-trade courses.

In assessing the implications of the above study, there are two equally important factors. Firstly, it appears that the students would not be able to solve practical problems which required the use of proportional reasoning without considerable assistance. Secondly, the post-secondary colleges would probably not be able to provide this assistance due to time limitations, the lack of mathematical background of the lecturers and the widespread acceptance of an algorithmic approach. It may be that the most appropriate strategy for teaching students to solve these proportionality tasks is to devise methods which are suitable for the particular situation, and which bypass the need for full proportional reasoning.

IMPLICATIONS FOR SECONDARY MATHEMATICS COURSES

The new Mathematics Our Way 8B text (Britt,1982) contains a good section on ratio. It covers scale drawing and enlargement, similar shapes, ratio as a comparison, simplifying ratios, percentage as a ratio, and a small section on practical problems. It provides a good introduction to the ratio concept. The grade 9 syllabus (Ministry of Education, 1978,a) focuses on the concept of rates and uses the practical examples of speed, pay rates, currency conversion rates, and conversions between measurement units. The grade 10 syllabus (Ministry of Education, 1978,b) mentions ratio as a comparison, writing ratios in simplest form and sharing, and dividing a quantity according to a given ratio.

This is a commendable emphasis to the development of the ratio concept. However, as is the case with most secondary courses,

there is too little emphasis on the application of the ratio concept to practical problems, and on the comparison of two ratios. It is for this reason that the students in this, and the Hart(1980,b) study did not recognise the proportionality questions as belonging to the type of problems which they had been taught. As was found in the Hart study, there were very few cases where students attempted to use "a/b = c/d" to solve these problems, even though each of the grade 10 teachers said that they had taught their students using this method.

It is important that secondary courses provide students with a wide range of examples which emphasise the essential aspects of proportionality, and particularly giving examples where it does and does not apply. It is appropriate that secondary programs teach strategies which are generally applicable. These strategies should be natural methods, based on the preferences of the students. A common strategy which students used in the study was to first determine a basic unit, and then to use the unit to calculate the answer. This is preferable to the "a/b = c/d" approach since students use this in an algorithmic way in the narrow range of classroom examples and do not apply it in the wider context. The natural methods both appeal more to an intuitive concept of proportionality and can be applied in the wide range of practical situations which the student will meet in their everyday lives.

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A PRELIMINARY REPORT ON THE USE OF CALCULATORS FOR PROBLEM SOLVING IN PAPUA NEW GUINEA PROVINCIAL HIGH SCHOOLS

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With the increasing availability of simple calculators, the growing acceptance of the use of calculators in many occupations and the current emphasis on the development of reasoning skills, it seems appropriate to investigate the possible advantages that the use of calculators might have in problem solving. In this intervention study, the writer gave a short program of instruction on the use of the calculator and strategies for problem solving with pre- and post tests to measure the change in performance.

HYPOTHESES

It was hypothesised that:

(i) the use of a calculator during instruction and practice of problem solving skills will increase subjects' ability to solve problems,

(ii) subjects will perform better on vocational type questions than on algebraic type questions,

(iii) the use of a calculator will develop confidence in problem solving skills.

SAMPLE

The program was administered to 206 grade 10 subjects in two Provincial High Schools in Lae, Papua New Guinea. Fifty five of these were females. Table 1 gives the sample distribution.

SCHOOL	SEX MALE	FEMALE	TOTAL
A	103	–	103
B	48	55	103
			206

TABLE 1: Sample Distribution

METHOD

The testing and teaching program consisted of six lessons. These included a pre test and a post test with four periods of instruction on the use of claculators and problem solving. The program was established as follows and was carried out over a three week period in each school:

Lesson 1: Pre–tests (i) Problems (ii) Basic number facts

Lesson 2: Introduction to the calculator and its use in computation

Lesson 3: The use of the automatic constant. The solution of routine problems

Lesson 4: The use of the memory keys. The use of an "estimate and check" strategy

Lesson 5: Revesion. Looking for patterns

Lesson 6: Post–test

The pre test consisted of fifteen problems including both algebraic and vocational content. The post test consisted of seven from these problems, four of which could be solved by the "estimate and check" strategy or by looking for a pattern, and three of which were vocational in content.

One class in each school completed the post test without a calculator. These calsses were told they need not complete the computations if they felt they were too time consuming or complex. In fact, very few, did not attempt to complete all calculations.

Although the lessons were planned to take 40 minutes, the time for periods in the second school varied from 30 to 40 minutes because of internal arrangements relating to Easter break.

For the four teaching periods, subjects were supplied with workbooks, adapted from material prepared for an National Council of the Teachers of Mathematics Conference, 1980.

RESULTS

(i) Number of Questions Attempted

One of the most significant results is that there was a significant (.001) increase in the number of questions attempted on the pre- and post-tests for the total population and also for the female subjects. The average number of questions attempted is given in Table 2. Only the seven questions common to both tests are included.

		Pre test/7	Post test/7
School A	Class 1	1.9	5.7
	Class 2	1.6	5.9
	Class 3	1.7	5.2
School B	Class 1	1.2	5.1
	Class 2	1.5	4.7
	Class 3	1.7	4.7
Total	School A	1.8	5.7
	School B	1.4	5.0
		t = 29.5	df = 204

TABLE 2: Average Number of Questions Attempted

(ii) Achievement

Similar increases in achievement scores took place on the part of all except two students in one school and approximately 75% of the other. The greatest increase was made by the older students.

Tests were scored as follows: 0 – non attempt
1 – partially correct
2 – correct

The increases which occurred are shown in Table 3.

		Pre test/7	Post test/7
School A	Class 1	3.9	27.5
	Class 2	3.2	22.3
	Class 3	5.0	23.8
School B	Class 1	6.8	29.5
	Class 2	2.6	13.5
	Class 3	2.0	7.0
Totals School A		4.0	24.5
	B	3.9	17.1
	Total	4.3	20.5
		t = 13.9	df = 205

TABLE 3: Average Percentage Achievement

The differences between pre test and post test scores is highly significant.

(iii) Types of Questions

The types of questions correctly completed by subjects is given in Table 4.

			Algebraic	Vocational
School A	Class 1		15	25
	Class 2	(B)	9	18
	Class 3		14	22
School B	Class 1		20	31
	Class 2		10	13
	Class 3		4	6
	Totals		72	115

TABLE 4: Types of Questions Correct

Vocational type questions appear to be easier for all students.

(iv) Other Results

(a) Girls in one class did better than the boys in the same class

(b) One class in each school did better than others

(c) It appears that the groups using the calculator for the post test had no particular advantage over those groups not using a calculator.

DISCUSSION

The significant increase in the number of questions attempted indicates a definite increase in confidence on the part of the subjects. That this increase was equally significant for those students not using a calculator in the post-test indicates that further consideration of the exact nature of that increase in confidence is needed. It may well be that the subjects' initial unfamiliarity with the investigator militated against full participation in the pre-test, though this seems a less likely explanation than that confidence was gained through the subsequent lessons and calculator use. What particular elements of the lessons contributed

to the increase in confidence is also uncertain. Elements such as the use of calculators, the instruction in strategies or the instructor's style of teaching are all possible factors.

The highly significant difference between scores on pre- and post-tests indicates that the intervening lessons had a positive effect on performance. Again, whether this was directly due to the use of the calculator or to the learning of specific strategies is not clear. That the classes using calculators for the post test did little better than those not seems to indicate that the learning of strategies may have been the dominant factor.

Grade 10 students appear to find problems of a vocational nature less difficult than those which are algebraic in type. This could be due to the perceived relevance of questions relating to postage, agriculture, etc. or to the lack of experience in algebra. The introduction of the "estimate and check" strategy was designed to overcome the subjects' lack of algebraic concepts.

On the post test, the highest score was 73% and this was shared by one female and four male subjects. Further analysis of sex differences is underway.

That the greater increases occurred among able students may have been due to the fact that these students were mainly in the classes which used calculators in the post-test. An attempt to further clarify this issue will be made by giving a retention test where approximately half these better students work without a calculator.

CONCLUSION

While no definitive conclusions can be drawn from this pilot study, there are indications that Grade 10 students improve their problem solving skills through the use of calculators. It is reasonably clear that they find vocational type questions easier than those of an algebraic nature.

The most important result seems to be the significant growth in confidence on the part of almost all students. Only six students did not attempt more questions on the post-test than on the pre-test. Only three attempted a fewer number of questions. This, together with the greater amount of working shown in answers, indicates definite growth in confidence.

AN OVERVIEW OF INVESTIGATION INTO THE DEVELOPMENT OF RATIONAL NUMBER CONCEPTS

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Work with fractions has long been a problem area in mathematics education. While complex operations with fractional numbers are no longer necessary, an understanding of fractional numbers is critically important as a means of handling continuous data, in the same way counting ideas can be applied to discrete data.

It is generally agreed that fractional numbers are complex by nature. Kieren (1979) puts forward the necessity to be able to operate with five sub-construct. These are part-whole, ratio, quotient, measure and operator. There is still no clarity as to how these sub-constructs develop, though previous research by Noelting (1978), Kieren and Southwell (1979), Karplus (1980), Noelting and Gagne (1980) and Southwell (1980, 1981 and 1982) on the operator and ratio sub-constructs have given some indications.

PREVIOUS STUDIES

(i) The study by Kieren and Southwell (1979) sought to trace the development of the operator sub-construct with a sample from Canadian elementary and secondary schools. Seventy two subjects in Grades 4 to 8 from public and Catholic schools were interviewed individually and asked to react to two different representations of the operator sub-construct. The two representations were a machine mode using a box with input and output and a pattern mode, with related pairs of numbers on cards. By these representations, unit and non-unit fractions,

direct and inverse fractions, single and composite fractions were tested. The following conclusions emerged:

(a) A subject who mastered the direct fractional number task was likely to master the related inverse task. It appears that it is the nature of the operator that determines performance rather than the aspect of inverse within the operator.

(b) A pattern of growth over time in the operator sub-construct occurs with 3 major levels:

½ operator
unit operator
general operator.

(c) "½" tasks were mastered better than all other tasks.

(d) There is a two year delay between mastery of the tasks using the two different representations.

(e) The difference between male and female performance on the machine mode was found to be significant at the 0.01 level.

(ii) Part of this study was replicated using a sample of seventy students from Grades 2 to 10 in Papua New Guinea. These subjects were from both urban and rural schools. The machine mode was used and treated in the same way as in the previous study. An orange juice test of proportionality developed by Noelting was also used. Similar conclusions emerged in relation to inverse tasks; a general pattern of achievement by age but with a greater delay than with the Canadian sample. Differences in performance between the sexes were also present.

Data analysed also indicate that the stages of proportional reasoning identified by Noelting (1978) do hold with the Papua New Guinean sample with some variations. The similarity between the two samples appears to increase as the stages develop. Differentiation in these stages does not appear to be determined by grade.

The strategies adopted by Papua New Guineans are similar to those of Canadian subjects at the pre-operational and concrete operational levels. No clear statement of strategies within the formal operational level is possible.

(iii) A study (Southwell, 1982) using an Australian sample indicates that Australian students develop the operator sub-construct of rational number in three general stages with a delay of approximately one year on Canadian subjects. Findings regarding inverse functions and composition reflect the Canadian results.

THE PRESENT STUDY

(i) Purpose of the Study

In order to extend the findings of previous studies and in pursuance of an ultimate goal of determining the most effective approach to the teaching of fractional numbers, further study was undertaken. The purpose of this most recent study was:

(a) to trace the development of the various sub-constructs with age

(b) to compare performance on different modes of representation of the rational number sub-constructs

(c) to analyse the mechanisms used by subjects in handling fractional number situations, and

(d) to examine implications for teaching rational numbers

(ii) The Sample

The sample consisted of 522 subjects from Grades 6, 8 and 10 in four schools in Lae, Papua New Guinea. The secondary subjects were from two different Provincial High Schools, both of which are mainly boarding schools. The sample distribution is given in Table 1.

Sex	Grades 6	8	10	Totals
Female	76	115	67	258
Male	81	101	82	264
Totals	157	216	149	522

TABLE 1: Sample Distribution

(iii) Instruments and Procedures

The first test administered was the Rational Number Thinking Test developed by Kieren who gave the investigator permission to use the test. Because of the relative unfamiliarity of Papua New Guinea students with pizzas, this was changed to "pie".

Kieren's test falls into four sections:

Items 1 to 6 have to do with mixing chocolate drink and is a form of Noelting's orange juice test. Items 7 to 14 involve sharing of pizzas (pies), giving a different representation of proportional reasoning. Items 15–18 involve the operator sub-construct and is a pencil and paper version of the concrete machine mode used by Kieren

and Southwell previously. The remaining items 19–24, present various problem situations in which the subjects are asked to perform rational number tasks. These emphasised the part–whole and quotient sub–constructs.

The second test used was a much shorter test, devised by the investigator to test the subjects' acquisition of the measurement sub–construct. Items 1 and 2 involve the division of a length into a number of equal parts and the designation of each division point by some fractional name. Items 3 and 4 involve the comparison of two fractions, while items 5 and 8 test the marking and reading of scales.

(iv) Results

Analysis of performance on these tests indicates that the tests involving ratio have produced comparable findings to previous studies. The section testing the operator sub–construct has proved more difficult as an abstract paper and pencil than as a concrete machine representation.

(a) Achievement by Grade

To trace the development of the various sub–constructs by grade, the means for each sex in each grade for each sub–test were computed. These are shown in Table 2.

On the Drinks, Share, Operator, Inverse and Scale sub–tests, there appears to be a gain in achievement by grade for both males and females. This pattern does not occur for males in grades 6 and 8 on the Problems sub–test nor with males and females on the measurement sub–test. Females in grade 8 and 10 on the inequalities test do not follow the pattern either.

Grade	Sex	Drinks	Share	Operator	Inverse	Measurement	Inequalities	Scale	Total
6	M	2.16	2.88	.11	.01	2.06	2.88	.18	.06	5.704
	F	1.85	2.47	.17	.03	2.17	3.12	.15	.04	6.1852
8	M	2.20	3.67	.63	.21	1.92	4.01	1.09	.59	6.2667
	F	2.27	3.01	.72	.17	2.45	4.66	1.08	.40	6.5625
10	M	2.82	4.23	1.5	.54	2.71	3.17	1.21	.85	7.000
	F	2.42	3.90	.97	.27	3.00	2.67	.90	.79	5.8750
Total	M	2.38	3.60	.74	.25	2.21	3.40	.84	.51	5.9778
	F	2.20	3.08	.63	.16	2.50	3.70	.76	.40	6.2549
Total		2.29	3.34	.69	.203	2.35	3.57	.80	.45	6.1250

TABLE 2: Means on Eight Sub–tests by Six and Grade Sub–tests

An SPSS crosstabs analysis, however, revealed significant differences betwen grade and male and female performance on some sub–tests. This is shown in Table 3.

	Drinks	Share	Operator	Inverse	Problems	Measurement	Inequalities	Total
Male								
x^2	17.66	43.21	10.47	1.01	19.44	65.64	23.90	6.82
df.	10	14	8	6	10	14	4	10
sig.	.06	.0001	.2330	.985	.0350	.0000	.0001	.7426
Female								
x^2	22.25	52.07	2.45	2.49	18.39	44.296	9.11	3.83
df.	10	14	6	4	10	14	2	8
sig.	.01	.0000	.8746	.6469	.0488	.0001	.0105	.8723

TABLE 3: Crosstabulation of Sub–tests by Grade Controlled by Sex Sub–tests

This indicates that the differences in mean performances between grades shown in Table 3 are not significant for both males and females on the operator, inverse and scale sub–tests.

Tests of significance show that the differences between the total means of grades 6 and 8 and grades 6 and 10 are highly significant, while the difference between the the total means of grades 8 and 10 is also significant, while the difference between the total means of grades 8 and 10 is also significant, though less so.

The significance of the gain between grades 6 and 8, however, needs to be treated with some caution since, in Papua New Guinea, only 60% of Grade 6 students proceed to secondary school. Grade 8 students, therefore, may well be of a higher standard because of the selective processes involved.

(b) Comparison of performance on different modes of representation

A correlation analysis was carried out to measure the strength of the relationship between the eight sub-tests. Significances are shown in Table 4.

	Drinks	Share	Operator	Inverse	Problems	Measurement	Inequalities	Total
Drinks	–							
Share	.001	–						
Operator	.001	.001	–					
Inverse	.001	.001	.001	–				
Problems	.001	.001	.001	.019	–			
Measurement	.054	.293	.007	.046	.001	–		
Inequalities	.001	.001	.001	.001	.001	.001	–	
Scale	.055	.016	.001	.001	.003	.004	.001	–

TABLE 4: Significances of Correlations between Sub-tests

The relationship between most sub–tests seems to be strong with the exception of the measurement and share sub–tests. Analysis of correlations by sex and grade does not yield such clear significances.

(c) Sex Differences

The previous result is supported by the fact that there were no significant differences between the total scores for males and females.

Further analysis is needed to ascertain differences in performance on each sub–test.

(v) Initial conclusions

Further analysis of data is continuing from the data presented, however, it seems that there is an increase in performance by grade, though no information is yet available as to the precise nature of that increase. Also there appears to be a strong correlation between performance in the sub–constructs measured by the tests administered.

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FROM ARITHMETIC TO ALGEBRA

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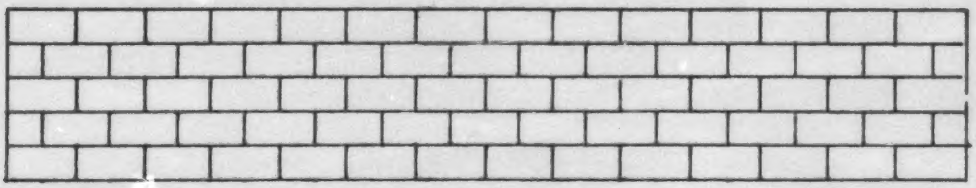
During recent years there has been much discussion regarding the teaching/learning of arithmetic but without much regard to the acquisition of the related algebraic skills necessary for anyone who wishes to involve oneself in the abstract thinking process called mathematics. For the most part, it seems that teachers present the various algorithms of arithmetic as a series of essential but meaningless steps and use the notions of commutativity, associativity, distributivity and so on as the unifying ideas that link arithmetic to algebra.

The research programme 'Concepts in Secondary Mathematics and Science' CSMS conducted in Britain between 1974 and 1979 has revealed a startling hiatus between, on the one hand, the teacher's expectation of the mathematical understanding acquired by her pupils and the mathematics actually acquired by those pupils. (It is also noteworthy that the findings of CSMS - at least in a general sense - have been duplicated in New Zealand's 'Learning in Science Project' LISP). CSMS offers such comments as, '. . . children for the most part, did not use teacher taught algorithms'. . . but rather adapt or replace those algorithms by their own methods'. LISP tells us that for the most part, children adapt and replace teacher taught scientific concepts by surface understandings that represent total misconceptions of the original ideas. CSMS goes further and contends that 'the teaching of algorithms when the child does not understand may be positively harmful in that what the child sees the teacher doing is 'magic' and entirely divorced from problem solving. One purpose of teaching mathematics in school is to provide children with the skills and information to solve problems. It is tempting therefore to attempt to hasten the acquisition of those skills by teaching a series of rules - conceptualised by ourselves as adults - and quite often not only meaningless to the child, but also presented in a manner that prevents the child from constructing any meaning

whatsoever. 'If to the child, the algorithms are not based on reality and to a large extent are seen as something separate and not necessarily logical, then we are certainly not fulfilling the purpose for which the teaching was intended.'

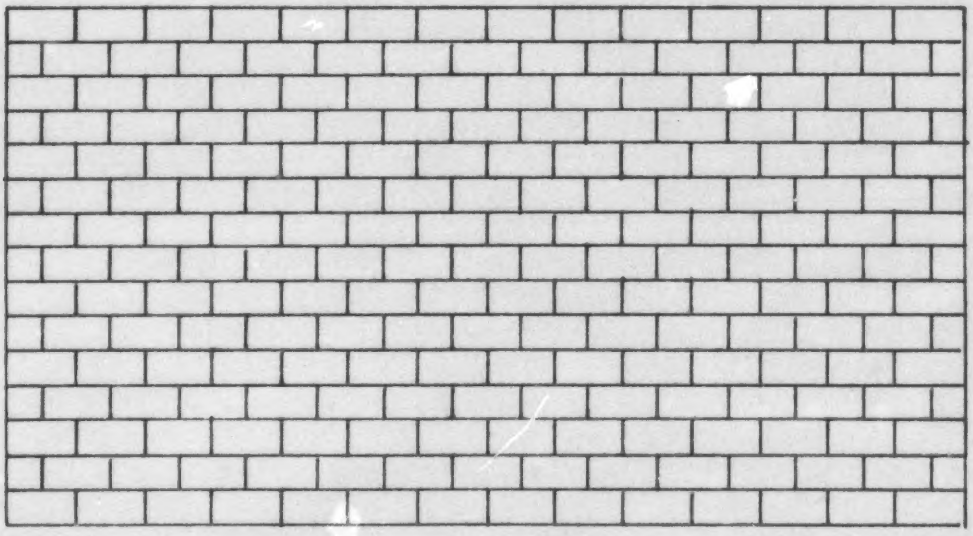
Suppose we ask a child 'how many bricks or blocks are in this wall?'

How many bricks in the wall?



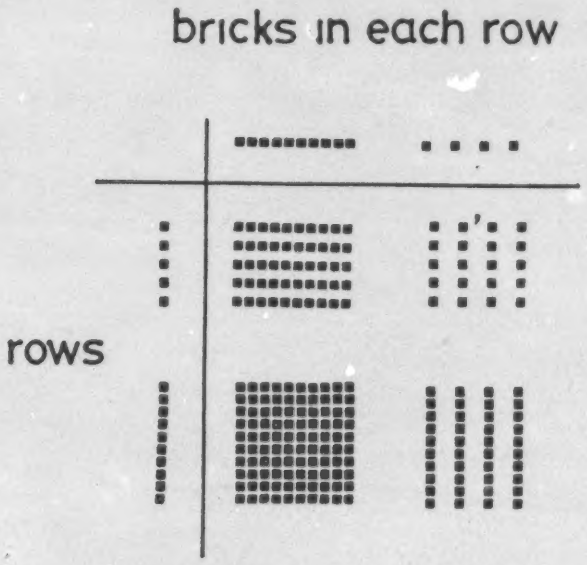
A first reaction will be to count them all, one by one, and that seems a reasonable thing to do. But suppose now, the wall looks like this.

How many bricks in the wall?

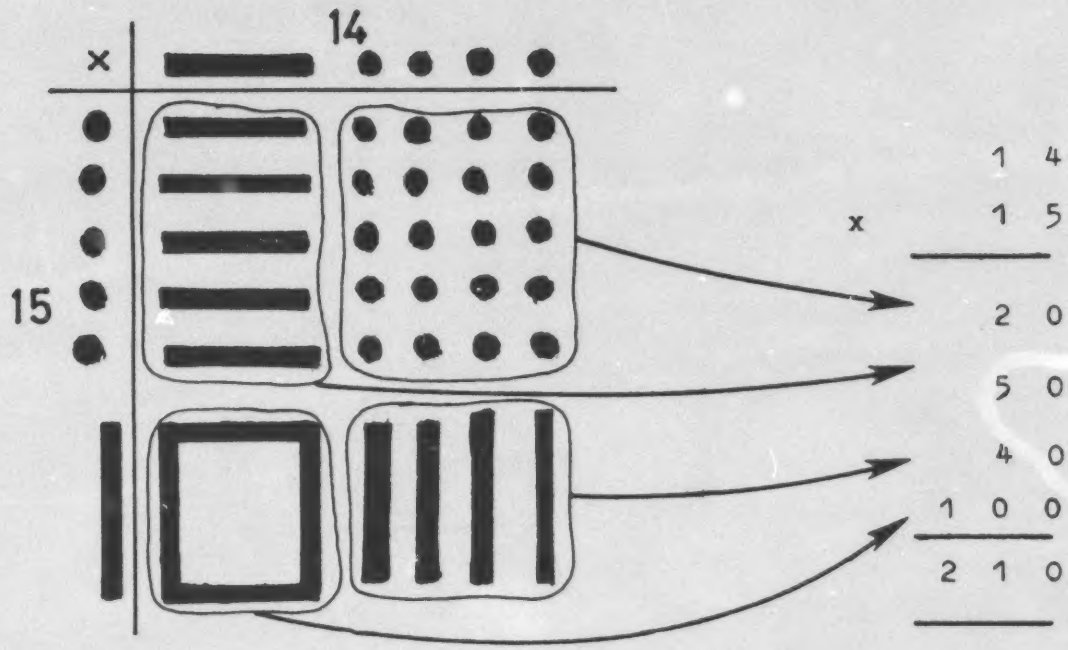


Again, it is possible to count them all one by one, but most of us - and that includes children - will look for a more efficient and economical method. (I tend to think of humans as problem solving animals with a lazy streak - always looking for our own short cuts). It's at moments such as this one that algorithms are born - born from reality.

Because our number system is a decimal one, it is convenient to 'model' the wall in this manner.



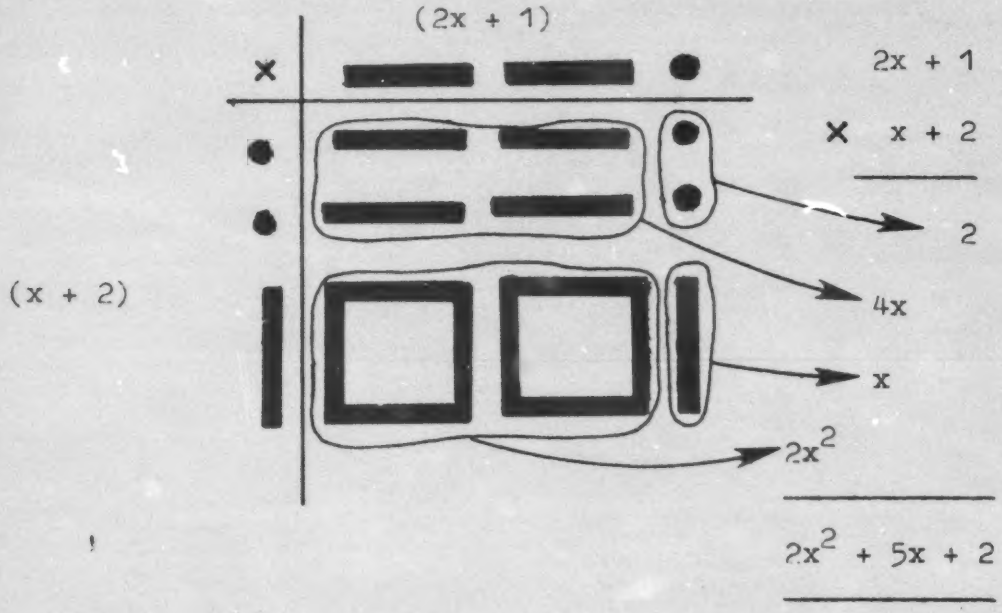
Moreover, we can follow the research findings of Bruner, Bishop and Lean who suggest that the internalisation of spatial ideas (most of our mathematics has its genesis in geometry) can be facilitated through a visualising process and in turn concepts that can be represented in visual form have a much greater chance of being internalised and so understood.



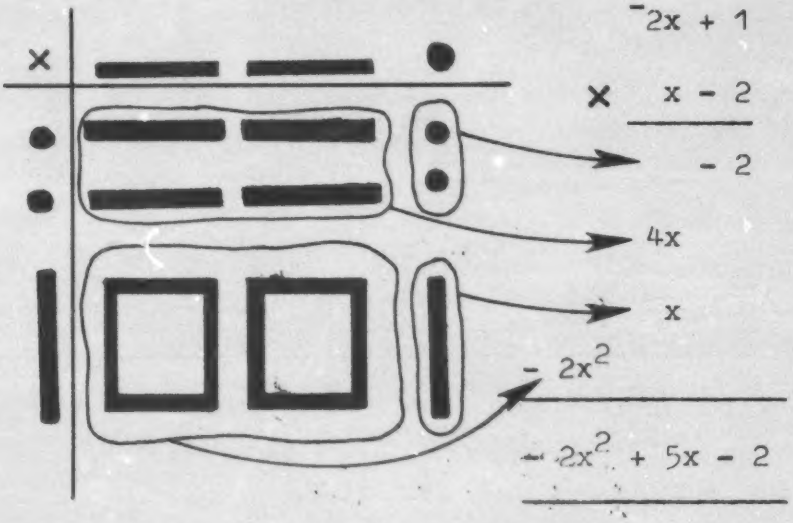
The product 210, can be read directly from the representation of the brick wall or it can be recorded as 4 separate partial products whose sum is 210. The symbolic algorithm - the sum of partial products - arises quite naturally from the pictorial representation of the brick wall itself. It is worth noting here that firstly, the process incorporates the distributive rule (it need not be taught as a separate rule) and that secondly, the algorithm can be shortened with little difficulty to the more traditional 3 step model. If that is everyone's wish, then so be it, but before you raise your hand in insistence, may I ask whether in these days when calculator is king, 'Is there a need to pursue pencil and paper, long multiplication ad nauseam?'

You may be wondering when the notion of algebra as a generalised arithmetic will appear. Right now.

Suppose we use ● to stand for 1 and ▬ to stand for x, this picture models the product (x + 2)(2x + 1).

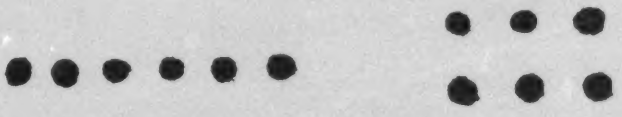


And further, if we use ● to stand for ⁻1, and ▬ to stand for ⁻x, then (x - 2)(1 - 2x) can be modelled by



These models, in turn, lead to the usual techniques required to expand such algebraic expressions but again it is noteworthy that the development uses the distributive rule without separating it as a formalised field axiom or piece of structural knowledge.

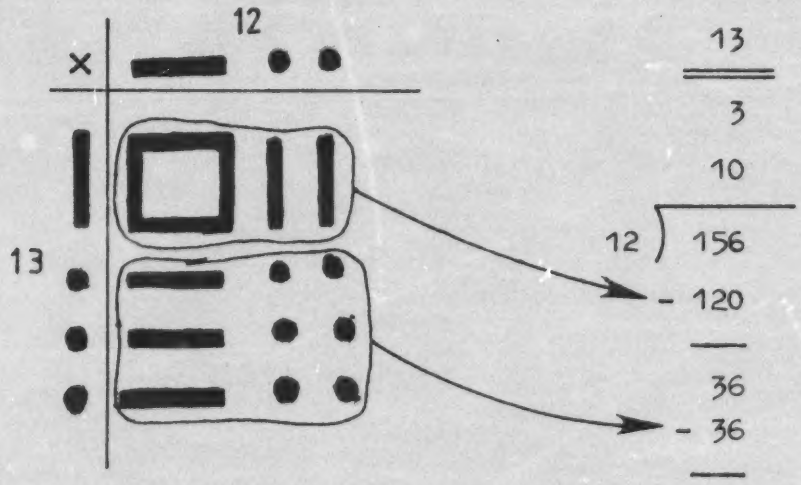
Piaget insists that for a child to be operational at an intellectual level as opposed to a perceptual level, the child must have acquired the notion of 'reversibility'. That is, if he can do an operation such as multiplication he must also be able to undo it - division. So as well as knowing that 2 groups of 3 is 6, the child must be able to re-arrange 6 as 2 groups of 3.



The division operation can therefore be concretised as the notion of forming rectangular arrays and is accordingly very closely linked with the process of factoring or factorising. If you like, division is finding a second factor having been given the first.

The operation 156 ÷ 12 can be interpreted as 'How many groups of 12 are there in 156?' 156 can be modelled as

We can respond to the problem by forming a rectangular array



$156 = (1 \times 10^2) + (5 \times 10) + 6$ and $12 = (10 + 2)$

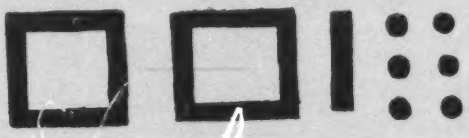
If we write $10 = x$ and $10^2 = x^2$, we get

$$(1 \times 10^2) + (5 \times 10) + 6 = x^2 + 5x + 6$$

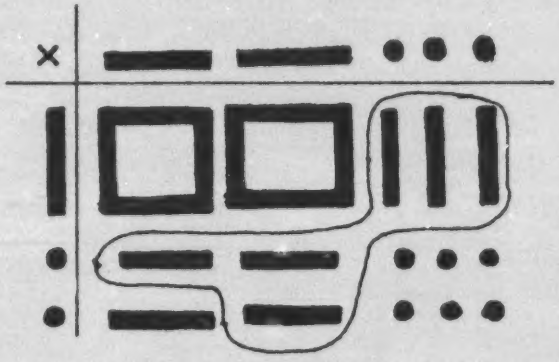
and $$(10 + 2) = x + 2.$$

Accordingly, the process of either factoring $x^2 + 5x + 6$ or of dividing this expression by $x + 2$, can be represented, as earlier, by the process of making a rectangular array. The array is of course the same as the one above.

To complete the picture, suppose one wanted to factor $2x^2 - x - 6$. This expression can be modelled as



The rectangular array that we can use to determine the factors of $2x^2 - x - 6$ is



So $2x^2 - x - 6 = (x - 2)(2x + 3)$

(It must be understood that together represent 0, a notion that needs deriving through an approach to operations with integers that considers $^{-}1$ and $^{+}1$ as discrete elements on the number line rather than the more usual shifts.)

Clearly, one doesn't expect the process of factoring trinomials to be forever the process of drawing rectangular arrays. The kind of deductive reasoning that is usually required - that of establishing rules associated with the constant term and the coefficients of both x and x^2 - can however be 'discovered' through the inductive reasoning process, that of drawing conclusions from a limited number of observations.

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'AVAILABILITY OF MATHEMATICS RESOURCES IN MOROBE PROVINCIAL HIGH SCHOOLS: A PRELIMINARY REPORT'

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ABSTRACT

Textbook writers often make basic assumptions on the availability of the resources at a school. This paper discusses the results of a survey carried out in Morobe Provincial High Schools on the availability of mathematics resources with particular reference to those prescribed in the 'Mathematics Our Way' books. It also presents the teachers' opinions on Mathematics Our Way books in general and proposes some areas for future research. The implications for inservice are also discussed.

INTRODUCTION

With the introduction of a new mathematics syllabus for Grades 7 and 8 in 1976, teachers expressed a need for a textbook to accompany the new syllabus (Robert, 1976). The Mathematics Our Way (MOW) books which initially were in booklet form were an answer to their request. Murray Britt, a UNESCO Curriculum Specialist, is the author of the MOW series books. He also helped with the designing of the syllabuses.

Grades 9 and 10 mathematics syllabuses were also prepared in the same format as those of Grades 7 and 8. This suggests that the Grades 9 and 10 versions of the text will take the same format as those of the earlier Grades. It is anticipated that Grades 9 and 10 texts will be available in schools by 1984 and 1985 respectively.

The survey discussed in this paper was a result of Godfrey Yerua's proposed study 'Development and Implementation of (Mathematics) Textbooks in Papua New Guinea' (Yerua, 1982). It was an attempt to provide Yerua with some additional information and to suggest some areas for possible inclusion in his study.

The concrete operational method of approach taken in the MOW books pre-supposes that the required resources are available in the schools. Because of the very method of approach taken in the textbooks, the successful teaching of any topic using the books will very much depend on the availability of the prescribed equipment/materials in the schools. The survey this paper reports investigated the availability of the prescribed equipment/materials, other teaching aids that are actually in the schools, and the attitudes of the teachers to the books and the instructional methods.

It makes suggestions for future research in this area to provide more information on the introduction of MOW books. Some of the questions posed when designing the survey were:

1. Do the schools have MOW 7A, 7B, 8A, 8B available in sufficient numbers?
2. What do teachers who use the MOW series in their maths lessons feel concerning:
   (a) the textbook itself,
   (b) the standard of the books,
   (c) the exercises in the books,
   (d) the problems with the books,
   (e) what improvements are needed,
   (f) whether the books are a useful resource.
3. Are the mathematics equipment/materials prescribed in the MOW series available in the schools?

4. If the equipment/material is not available, what does the teacher use as a substitute?

5. What other equipment/materials and textbooks are available in the schools?

6. Are there any mathematical games available in the schools?

METHOD

The survey was conducted by means of a questionnaire which was sent out to eight of the Provincial High Schools in the province. Wasu Provincial High School, a new school opened only in 1983, was not surveyed. Over twenty questionnaires were sent out to them, about three to each school. The questionnaire was directed to the mathematics teachers who are assumed to have used the MOW books themselves in their lessons. Only eleven replies were received so the data was less than anticipated, but the sample was representative since there was at least one reply from each school.

The questionnaire had five sections. They were:

1. MOW Textbooks
2. Resources
3. A.V. Equipment, Materials and Aids
4. Mathematical Games
5. General Questions

Using these five sections conveniently divided the questionnaire into useful areas for discussion.

PRESENTATION OF RESULTS

The method of presentation tries to present as much as possible the teacher's actual responses on the questionnaire. Since the size of the sample is small, this enabled the use of such an illuminative procedure.

The questionnaire used in the survey is shown in the appendix.

1. MOW Textbooks

When the teachers were asked to quote the number of MOW 7A, 7B, 8A, 8B they had available in their schools, the responses ranged from 11 to 210 being available.

The responses quoted below reveal the attitudes which teachers had to these MOW textbooks.

Question (g):
Do you find the 'Mathematics Our Way' books useful in your teaching? Circle one or more of the following statements as your answer.

(In each case, the numbers in brackets at the end of each statement indicate the number of respondents who selected that particular statement).

(i) *I find the MOW books not at all useful in my teaching.* (0)

(ii) *I find the books useful sometimes but are not very necessary resource.* (1)

(iii) *I find the books very useful in my teaching only for certain topics.* (5)

(iv) *I find the books very useful in every aspect.* (6)

(v) *Other - please state:* (3)

Comments stated under *(v)* were:

- Yes, but not really because most of the things are sort of baby stuff. Probably useful for primary school.
- As an aid, not a book to replace teacher.
- I find some exercises very useful.

Question (h):
What problems, if any, did you have or notice when using the MOW books in your teaching?

Circle one or more of the following statements as you answer.

(i) *Exercises in the books are sometimes too difficult to understand.* (1)

(ii) *Some topics are not covered adequately in the books.* (4)

(iii) *The books aim too high.* (0)

(iv) *The books aim too low.* (6)

(v) *There are too many exercises in the books.* (0)

(vi) *There are too few exercises in the books.* (2)

(vii) *Other - please state:* (4)

The following comments were stated under *(vii)* for this question:

- I'm particularly confused about the continual use of tables for x and ÷ and thus a slight alteration of the old methods.
- Particular approaches to a topic are sometimes confusing (eg. long division)

- Examples are confusing.
- There are some exercises which shouldn't be there at all.

Question (i):
What improvements, if any, would you like to see made to these MOW books?

Circle one or more of the following statements as your answer.

(i) *Number of exercises in the books should be increased.* (4)

(ii) *Number of exercises in the books should be reduced.* (1)

(iii) *The books should in general be upgraded to a higher level.* (8)

(iv) *The books should in general be lowered to a lower level.* (2)

(v) *The books do not need any improvements.* (1)

(vi) *Other - please state:* (3)

Comments stated under *(vi)* for this question were the following:

- Generally there are sufficient exercises aimed at the proper level. I'm mostly concerned with certain methodology (place-value methods of multiplication and dividing)
- I would like to see more exercises done for topics students find difficult so that they can spend more time on them.
  Eg. Fractions, decimals and percentages.

. The exercises are alright except that there are too many simple or easy things which take up more time like examples and trying to explain a concept.

Question (l):
How do your students find the MOW books?

Circle one of the following statements as your answer.

(i) *The books are too difficult for students to read and comprehend.* (0)

(ii) *The books are too difficult for students to understand.* (0)

(iii) *The books aim too low and therefore not very challenging for students.* (6)

(iv) *The books aim just at the right level of students' understanding and comprehension.* (4)

(v) *Other - please state:* (1)

The comment stated here was:

. Need more harder problems or challenging questions at the end of each topic.

Question (m):
How do you as a teacher view the MOW books?

Circle one or more of the following statements as your answer.

(i) *The books are very useful resource material for both staff and students.* (7)

(ii) *The books are very necessary for an effective teaching/learning process.* (2)

(iii) *The books are <u>not</u> always helpful.* (0)

(iv) Topics in the books can be better taught without the use of 'MOW' books. (2)

(v) The books should not be used at all. (1)

(vi) Other - please state: (3)

These were the comments stated under (vi):

- MOW are used only for exercises. There are few topics which can be taught without MOW.
- The books are a useful resource material to students because they can do a lot of work from them in their own time. Makes job easier for teachers. They give worksheets from the books rather than having to run off worksheets or write them on the board.
- Students follow up well with the use of the book. They have seen the topic and have read it before the teacher explains it in class. Sometimes it is not helpful when we don't have materials on the particular topic.

2. Resources At School

In the questionnaire, teachers were provided with a list of prescribed equipment/materials in the MOW books and were asked to indicate whether a particular item was available (YES) or not available (NO). (see appendix).

The items which can be considered as being generally not available in the schools are as shown in table 1.

ITEM	NO. OF TEACHERS WHO INDICATED AS NOT BEING AVAILABLE
Coloured pens	7
Paints	6
Squared paper	4
Playing cards	6
Sticky tape	4
Weighing scales	4
Measuring cylinder	4
Measuring tape	5

TABLE 1: Items not available in schools.

Those items which can be considered as being generally available are shown in table 2 below.

ITEM	NO. OF TEACHERS WHO INDICATED AS BEING AVAILABLE
Scissors	9
Glue	8
Dice	7
Newspapers	10
Trudle wheel	9
Spring balance	7
Centicubes	9
Protractors	10
Graph paper	7
Set squares	8
Compasses	9
Rulers	9
Meter rulers	9

TABLE 2: Items available at schools.

It must be noted here that of the items available at the schools, some of the respondents actually stated the numbers available and in many cases being less than half a dozen. So items, even if only 1 available was taken as being available.

They were also asked to write down what they use as a substitute in the absence of the required equipment/ material. Their responses varied a lot but the common ones were that they either did not do the activity which requires the use of the missing item, or made something similar; for example make measuring tape by cutting up strips of graph paper and sticking these strips together.

Teachers were also asked to select items from the list which they considered most useful and very necessary when using the MOW books. Among the items selected were those shown in table 3.

ITEM	NO. OF TEACHERS WHO SELECTED THE ITEM AS USEFUL
Glue	5
Squared paper	6
Trudle wheel	4
Weighing scales	4
Protractors	7
Compasses	6
Rulers	8
Meter rulers	4

TABLE 3: Items considered most useful and very necessary by teachers, when using MOW books.

Two items clearly overlap in tables 1 and 3. These are squared paper and weighing scales.

3.(i) A.V. Equipment, Materials and Aids

Items referred to in this section are assumed to be used in general and not necessarily with the use of the MOW books. Here, teachers were provided with a list of items and were asked to indicate with YES/NO if available or not available. They were also asked to indicate if the particular item is used in mathematics

as a teaching aid. From their responses, the items which can be considered as being generally available in the schools and used in mathematics classes are as shown in table 4.

ITEM	NO. OF TEACHERS WHO INDICATED AS BEING AVAILABLE	NO. OF TEACHERS WHO INDICATED ITEMS BEING USED IN MATHS
Overhead projector	6	–
Slide projector	9	–
Cassette recorder	9	–
Cassettes	7	1
O.H.P. Transparencies	6	–
Pictures	8	4
Charts	10	8
Calculators	8	3

TABLE 4: Items available in schools and their use in maths.

(ii) Other Textbooks

Teachers were asked to list other textbooks they had in in their schools apart from the MOW books. The common ones among those listed were the following:

- Modern Mathematics 1 & 2
- On Course Mathematics Forms 1–4
- Introduction to High School Mathematics
- Keep Mathematics Fit (KMF)
- Basic Mathematics 1 & 2

4. Mathematical Games

One way of developing mathematical concepts is through mathematical games. This section was included to see what is available in terms of mathematical games.

Teachers were provided with a list of twenty games and were asked to indicate on the list which games they had available in their respective schools. Their responses show that one of the few games widely available in schools is 'TANGRAMS'. This was indicated by five of the eleven

respondents. One or two schools had no more than three games and in some cases nil.

One of the interesting features of this section on mathematical games was when they answered the following question.

Do the games you have, adequately cater for your needs?

They were required to answer YES or NO and the responses were:

YES	NO	(Two did not
5	4	answer this question)

5. General Questions

In this section, teachers were asked to give a listing of mathematics resources, equipment and etc. other than those covered in the earlier sections of the questionnaire, which were not available in their department but which they should have. Teachers made a listing of items some of which they already indicated in the earlier sections as not being available.

Among the common ones listed were:

- Reference texts for teachers
- Calculators
- Protractors
- Set squares
- Compasses
- Scissors
- Dice
- KMF for Grades 8–10
- Floor scales
- Beam balances
- Coloured pencils

One of the questions asked in this section was:

What special resources, if any, are required by you as a maths teacher? List them.

The response here was 'MOW, Teachers' Edition', which simply means that some teachers have not yet received them.

SUMMARY

At least two schools have not got enough MOW books (one school had only 11 MOW 7A). In such a compact province as Morobe where the schools are relatively easy to get to, all schools should be adequately stocked with books.

The comments and responses from teachers on the standard of MOW books clearly indicate that:

(i) MOW books should be upgraded to a higher level

(ii) more exercises should be included in the books

(iii) MOW books are a useful resource for the schools

(iv) certain methods of approach in the MOW books are confusing to some teachers.

The comment to be made here is that teachers, despite their criticisms of the MOW series books, regard them as a useful resource material for them and their pupils. This can be taken as a positive attitude towards the MOW books. Teachers are, in fact, the best people to comment on any curriculum material as they are the ones who actually implement it and make direct observations on the progress of their pupils.

Some of the teachers' comments on the MOW books do support the points raised by Lawrence Ros on his paper 'A Critique on Randall Souviney's Approach to Teach Number Facts In Grade 7' (Ros, 1982). If teachers have difficulty understanding the new methods used in the MOW books then one would assume that there is very little chance that their pupils will understand them.

Perhaps curriculum planners and administrators should pay particular attention to the views of teachers who play an important role in the implementation process of any such curriculum material. The successful implementation of the MOW books in schools could well depend on how teachers react to them.

It is inevitable, that the introduction of any new material such as the MOW books into the schools must face some opposition. But the fact is that the Papua New Guinea High Schools now have a textbook which never existed before. It may serve as a starting point from which to develop and upgrade on a continuing basis.

Some equipment/materials are available and others are not. Of those that were not generally available were squared paper and weighing scales which were among the items teachers indicated as being important when using the MOW books. Here again, the effective use of the MOW series books will largely depend on the availability of the prescribed materials and so it is important to ensure that these are available in schools in sufficient quantities if they are to be of any use. It is worth stating here, that the successful implementation of the MOW textbooks will depend on the support schools receive from the Department of Education in the supply of the basic equipment/materials.

General teaching aids can be quite useful if teachers can use them well. They are especially useful for concept development and such other associated learning/teaching processes. Attempts should be made to introduce a greater variety of teaching aids into the schools.

From the list of the extra textbooks available in schools, it is obvious that most of them are the remains of the old Australian textbooks from the pre-independence period. These were the texts that were used with the

old Australian type syllabus at the time. Schools could be encouraged to purchase new and more modern texts which would also enable them to keep phase with developments overseas. It is also important for both teachers and their pupils to have access to textbooks other than the MOW series.

There are very few mathematical games, if any, are available in the schools. The non-availability of games in Morobe Provincial High Schools may indicate a general attitude of teachers' to games. It would be interesting to investigate the teachers' opinions on mathematical games in the future.

Teachers being ill-equiped with the basic resources will not be able to perform their duties as well as they should. This is particularly important in this country where there is so much outcry on low standards in education.

FUTURE RESEARCH

This is one province, MOROBE, what about other provinces? Clearly, a national view must be formed.

Future research using a larger sample could:

(i) investigate the extent of inconvenience caused to both teachers and their pupils due to the introduction of the MOW series in terms of

  (a) not enough MOW textbooks

  (b) lack of essential equipment and materials

(ii) investigate teachers' opinions on the MOW series and the extent of the book's use by both the teachers and their pupils.

(iii) investigate the achievement levels of students with regard to availability and non-availability of the required equipment and materials.

(iv) Pupil progress (or achievement) in mathematics in the community schools depend on the availability of classroom materials, location of the school and methods of teaching (Bob Roberts, 1981).

Does a similar pattern exist in the provincial high schools?

IMPLICATIONS FOR INSERVICE

It is clear from the teachers' comments and responses on the questionnaire at least for Morobe Province, that teachers need assistance to be able to use the MOW books effectively. That is if they are to use the books to their advantage rather than the books controlling the teachers' method of approach in teaching any particular topic. The teacher should be the best judge on what his/her pupils should do and should not do in so far as the use of the books are concerned.

Inservices should be arranged to help teachers overcome some of their difficulties in using the MOW books. Such inservice should aim to:

(i) enable the teacher to decide which sections of the textbook are suitable for his/her particular class and which ones are not.

(ii) help teachers understand some of the new approaches and methods used in the books.

(iii) help the teacher cope with the situations where required resources for teaching of a particular topic are not available.

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APPENDIX

SURVEY OF MATHEMATICS EQUIPMENT/MATERIALS IN PROVINCIAL HIGH SCHOOLS

Questionnaire on how teachers in Provincial High Schools view resources.

SECTION 1: Text Books

(a) Does your school have 'Mathematics Our Way' books?

Yes ☐ No ☐

(b) If YES, complete the table below.

'Mathematics Our Way' Book:	Total No. in School
7A	_____
7B	_____
8A	_____
8B	_____

(c) Are there enough of each in your school?

Yes ☐ No ☐

(d) If NO, which ones are in short supply?

List them here .....

(e) Is there enough for one to each pupil or one between two pupils?

Yes ☐ No ☐

(f) Can the student take it home?

Yes ☐ No ☐

(g) Do you find the 'Mathematics Our Way' books useful in your teaching?

Circle one or more of the following statements as your answer.

(i) I find the MOW books not at all useful in my teaching.

(ii) I find the books useful sometimes but are not very necessary resource.

(iii) I find the books very useful in my teaching only for certain topics.

(iv) I find the books very useful in every aspect.

(v) Other - please state:

(h) What problems, if any, did you have or noticed when using the 'MOW' books in your teaching?

Circle one or more of the following statements as your answer.

(i) Exercises in the books are sometimes too difficult to understand.

(ii) Some topics are not covered adequately in the books.

(iii) The books aim too high.

(iv) The books aim too low.

(v) There are too many exercises in the books.

(vi) There are too few exercises in the books.

(vii) Other - please state:

(i) What improvements, if any, would you like to see made to these 'MOW' books?

Circle one or more of the following statements as your answer.

(i) Number of exercises in the books should be increased.

(ii) Number of exercises in the books should be reduced.

(iii) The books should in general be upgraded to a higher level.

(iv) The books should in general be lowered to a lower level.

(v) The books do not need any improvements.

(vi) Other – please state: .....

.....

.....

(j) Are there any topics in the 'MOW' books which you would like to see upgraded?

YES ☐ NO ☐

(k) If YES, which ones? List them .....

.....

.....

(l) How do your students find the 'MOW' books?

Circle one of the following statements as your answer.

(i) The books are too difficult for students to read and comprehend.

(ii) The books are too difficult for students to understand.

(iii) The books aim too low and therefore not very challenging for students.

(iv) The books aim just at the right level of students' understanding and comprehension.

(v) Other – please state:

(m) How do you as a teacher, view the 'Mathematics Our Way' books?

Circle one or more of the following statements as your answer.

(i) The books are a very useful resource material for both staff and students.

(ii) The books are very necessary for an effective teaching/ learning process.

(iii) The books are <u>not</u> always helpful.

(iv) Topics in the books can be better taught without the use of the 'MOW' books.

(v) The books should not be used at all.

(vi) Other – please state:

.....

.....

.....

SECTION 2: Resources At School

When preparing the 'Mathematics Our Way' books, the basic assumption is that the materials required under each topic are available at the school. The table below is intended to collect information on the availability of the materials required when using the 'Mathematics Our Way' books by the Provincial High Schools.

(a) (i) Complete the table.

	ITEM	Is it available at school? (Yes/No)	If No, what do you use as a substitute?
Eg. 1	Cardboards	Yes	
2	Magazines & etc.	No	Newspapers
	coloured pens		
	paints		
	scissors		
	glue		
	dice		
	squared paper		
	playing cards		
	sticky tape		
	newspaper		
	trudle wheel		
	spring balance		
	centicubes		
	weighing scales		
	measuring cylinder		
	protractors		
	graph paper (cm squared)		
	set squares		
	machine nuts & bolts		
	compasses		
	rulers		
	meter rulers		
	measuring tape		

(ii) Add any items which may have been left out from the table and completing the table in the same way.

(b) Which of the resources (from the above list) in your opinion are most useful and very necessary when using the 'MOW' books? Underline them on the list.

(c) Which resources are least needed? Put a circle around them on the list.

(d) Would you consider any of these resources listed as being too expensive?

YES ☐ NO ☐

(e) If YES, put a cross (x) beside each item on the list above which you would consider as being too expensive.

(f) For those items which are adequately supplied to your school, is sufficient use being made of them by the staff?

YES ☐ NO ☐

(g) If NO, state the reason why: .....

.....

.....

(h) Are you familiar with the material covered in the MOW books?

YES ☐ NO ☐

(i) If NO, which parts or topics are you not familiar with? List here .....

.....

.....

SECTION 3: A.V. Equipment, Materials & Aids

In this section, the items referred to are assumed to be used in general and not necessarily with the use of 'MOW' books.

(a) (i) Complete the table below.

ITEM	Is it available at school? (Yes/No)	Is it used in maths? (Yes/No)	Do you use it? (Yes/No)
16 mm projector projector			
Over Head projector			
Slide projector			
Cassette Recorder			
Slides			
Cassettes			
O.H.P. Trans-parencies			
Pictures			
Charts			
Calculators			

(ii) Add any items that you have in your school and not included in table above – complete table in the same way.

(b) Are there other textbooks available at your school besides the 'Mathematics Our Way' books? Yes ☐ No ☐

(c) If YES, what are they? Fill in the table below.

Title of Textbook	Is it used by teacher only? (Yes/No)	Can the book/s be used by students (Yes/No)	If yes, students of what grade can use the book/s?
Eg. Modern Mathematics 2	No	Yes	Grades 9 & 10

SECTION 4: Mathematical Games

(a) (i) Here is a list of Mathematical games, complete the table.

(ii) Add to the list any games you have at your school which are not included in the list - complete table in the same way.

Title of Game	Is it available at school? (Yes/No)	If yes, is it available for students to use? (Yes/No)
Mancala		
Mastermind		
Spiro		
Sapce Lines		
Nine Men's Morris		
Tri-Ominoes		
Magic Squares		
Go		
Krypto Cards		
Jiggle		
Solitaire		
Tuf		
Equable		
Hindu Pyramid Puzzle		
Othello		
'L' Game		
Tangrams		
Mod Eight		
Epsilon		
Sequence Blocks		

(b) Do the games you have, adequately cater for your needs?

Yes ☐ No ☐

(c) If NO, what additional games would you like for your school? List them here .....

.....

(d) How could these be obtained?

SECTION 5: Some General Questions

(a) Are there any resources missing from your department which should be there?

Yes ☐ No ☐

(b) If YES, what are these? List them.

.....

.....

.....

(c) What additional resources would you like to have in the school? List them.

.....

.....

.....

(d) How can these be obtained?

(e) What special resources, if any are required by you as a maths teacher? List them.

.....

.....

.....

PROBLEM SOLVING: A REVIEW OF THE LITERATURE

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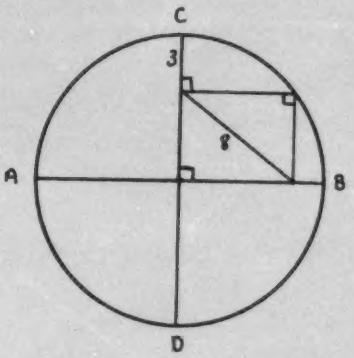
ABSTRACT

Problem solving has been an area of interest for educationists for a long time. In particular it has stimulated much research in mathematics education from primary level schooling through to tertiary studies for a number of years. This review will forcus on the results of research contained in the literature which has been focussed at the top of schooling and on into tertiary education. It is hoped that the review will give some sort of perspective to the following discussion and forum on problem solving here at the University of Technology.

PROBLEM SOLVING: A REVIEW

Jeremy Kilpatrick, a leading authority on problem solving in U.S.A. presents two points which I would like to share with you. The first is the title of a paper he wrote? "Stop the Bandwagon, I Want Off". In it he cautions us mathematics educators to go beyond the slogan which problem solving seems to have become for the 1980's to the substance of problem solving itself. The second is in a paper written in response to the well publicised "Agenda for Action" which advocates problem solving as "the foucs of mathematics for the 1980's". In this, Kilpatrick makes the point very strongly that one needs to know what is meant by the word "problem". This point is also raised by Lester (1980) in writing about research in mathematical problem solving. He used the following illustration:



"AB and CD are diameters of the circle.
How long is the radius of the circle?

Possible reactions to this question include the following:

1. I'm not much good at geometry, I can't be bothered.
2. It's a trick question. Why bother?
3. Interesting! I must think about this sometime.
4. Now, let's see! If I draw a diameter paralled to the segment which is known to be 8 units, then....

5. That's easy! The radius is obviously 8 units, since the given diagonal of the rectangle is 8 units and the other diagonal is a radius.

6. I've seen this before. The answer is 8 units".

Both Lester and Kilpatrick (Lester, 1980) make the point that problem solving is a very personal type of activity, in which our success is influenced by previous experience, our mathematical knowledge, the problem structure and the level of our interest and motivation. Kilpatrick further analyses the definitions given in a collegiate dictionary in which five senses of the word "problem" are given as follows:

"1a. a question raised for inquiry, consideration or solution.

b. a proposition in mathematics or physics stating something to be done.

2a. an intricate unsettled question.

b. a source of perplexity, distress or vexation"..

(G. and C. Merriam, p. 17).

The first of these senses is the more objective and external, while the second is more subjective and internal. The first emphasises the sense which suggests that a problem is something that someone gives you while the second sense depends on you as an individual. We seldom recognise the distinction between these two senses in our teaching, nor do we always recognise that success in problem solving depends on an acceptance of the problem by the potential solver. If, however, we carry this subjective sense of a problem too far, we could have everyone in this room convinced of the importance of problem solving, yet have as many different ideas of what that means.

To restrict that possibility slightly, we shall use this definitions as the basis for our discussion. "A problem is a complex task, usually expressed in words, that is accessible to the student, that is, he/she is capable of understanding the task and performing the operations needed for a solution but for which he has not learned an algorithmic procedure". This incorporates the three necessary elements for a question or task to be a problem, viz (a) a goal is to be reached
(b) an obstacle prevents ready solution
and (c) the student is motivated to find a solution.

It follows that problem solving is what we do to perform the task. It's importance is self evident if we thoughtfully consider the exigencies of every day life. This importance is reflected in the stance taken by several organisations and curriculum projects around the world. The one which has probably roused the most discussion is that of the National Council of the Teachers of Mathematics which has issued the statement "An agenda for action for school mathematics for the 1980's". In the eight recommendations made, the first is that "problem solving be the focus of school mathematics in the 1980's".

This emphasis resulted from the PRISM Project (Priorities for School Mathematics) in which 2,500 providers and consumers of mathematics education were surveyed.

In keeping with this emphasis, the Australian Mathematics Education Project adopted a similar emphasis. Although with the demise of the Curriculum Development Centre which funded it, AMEP's future became uncertain, it has continued to promote various aspects of problem solving.

The Scope of Problem Solving Research

It is a helpful exercise to categorise the variables inherent in a problem solving situation. In this way, it is easier to isolate the effects of each of these variables on the process of problem solving. Again the categorisation given by Kilpatrick (1978) is a useful starting point. Table 1 gives this categorisation.

Table 1

Kilpatrick's Categories of Problem Solving Research Variables

Independent Variables

Subject Variables
- Organismic Variables
- Trait Variables
- Instructional History Variables

Task Variables
- Context Variables
- Structure Variables
- Format Variables

Situation Variables
- Physical Setting
- Psychological Setting

Dependent Variables

Produce Variables

Process Variables

Evaluation Variables

Concomitant Variables

(Kilpatrick, 1975)

The three main categories of independent variables – subject, task and situation variables – are derived from the necessary components in any problem solving situation. These are the problem solver (subject) solving a problem (the task) under a set of conditions (situation).

Briefly, subject variables are quantities which describe specific attributes of the subjects. Organismic variables are such attributes as age, sex, socioeconomic status, geographic residence. Most studies on problem solving consider only those attributes of age and sex. Trail variables seem to be of more interest. These include attitude, cognitive style, mathematical memory and persistence. Krutetskii (1976) outlined the structure of mathematical abilities using traits in this sense. Instructional history variables describe mathematical topics studied, school attended and problem solving strategies developed. These are hardly ever considered in research on problem solving.

Task variables are those which describe the problem task. Context variables describe the physical situation of the problem, as well as the language in which the problem is written. Structure variables describe the internal structure of the problem while format variables indicate the different ways a problem may be presented.

Situation variables are those which describe the physical, social and psychological environment in which the problem solving occurs. Physical variables include the type of space to be used, the nature of the space and the resources available to the subject. The psychological variables describe the purpose of the problem solving experience (instruction, practice, testing), the type of procedure and the nature of the learning environment (type and amount of feedback, etc). Kulm (1978) also includes social variables in this section. By these, he means

such aspects as the size of the group, and the interrelationships which exist between subjects and between subjects and experimenter.

The dependent variables which form the second major group of variables in problem solving research are the result of the subjects' responses to the problem task. Product variables describe achievement on the problem task and include the time taken, correctness or otherwise of the solution and the completeness of the solution. Usually speed and accuracy are the only two considered. Process variables describe the processes used by the subject, the algorithms used and false starts made by the subject. The common methods of obtaining information on these variables are by verbal reports of subjects – a method which has certain inherent dangers – from written work or from observation of manipulation of material. Evaluation variables are those which express the thoughts of the subjects upon completing the problem including level of confidence in correctness of the solution, and how the problem was seen in relation to other problems. This is an area in which it is relatively difficult to obtain reliable data. Concomitant variables are described as those variables not included in the previous three groups but which may still change in the course of the problem solving experience.

More attention seems to have been given to problem solving research than to research on any other topic in the mathematics curriculum. Despite this, there is no widespread agreement even about what are the most important issues to investigate. Reviews of the research have appeared from time to time, one of the most recent being that of Lester, (1980) in which he summarised some of the research completed in the four categories already listed. Research involving primary and lower secondary subjects seems to be more prevalent than with older subjects, though it is recognised that many conclusions

arrived at with younger subjects apply equally well with older subjects.

The international emphasis on problem solving can be seen in projects that have developed in U.S.A., Canada, England, U.S.S.R. and Australia. Time does not permit more than a cursory mention of most of these projects but the fact that they are so wide spread speaks for itself.

Projects such as Mathematical Problem Solving Project which relies on retrospection are becoming more prevalent.

Task Variables

That there is no general agreement on certain aspects of problem solving could well be due to the diversity of problems used in research. These range from questions requiring direct computation to puzzle type questions. Lesh (1979) maintains that types of problems used in research have been primarily the simple story problem and puzzle types. This implies that research has not given attention to problems with rigorous mathematical content into realistic situations.

There seems to be some agreement that performance on a problem task is influenced by the task itself. This has led to a number of studies that focus on characteristics of problem tasks including the mathematical content and method of representation. The structure of the problem is another characteristic which has received some attention. So far no sets of variables has emerged that could be claimed as determining success on problem tasks.

One area of particular relevance to Papua New Guinea is that of language and the ability to read. There is, in fact, conflicting evidence about the ability to read and its effect on problem solving performance. The difficulty level of the syntax was found by Linville (1970) to have

an effect on problem solving. But Knifong and Holtan (1976) say that their evidence is that computational errors are the most significant hindrance to success. Jerman (1973, 1974) found that linguistic predictor variables are grade or age related. He also claims that the significance of computational variables as predictors decreases from being very significant with intermediate age students to having little value at all with college students. (Jerman, 1974)

Caldwell (1977) designed a study to determine the relative difficulty of different types of word problems at different ages. Certain factors such as length, syntactic structure, vocabulary level were kept constant while the two dichotomous variables were concrete versus abstract, factual versus hypothetical. Elementary subjects solved more hypothetical than factual problems across all grades, the abstract problems were more difficult than the concrete ones. For all grades, too, the concrete/factual problems were the easiest. "There was a significant interaction between the level of abstractness and the degree of factualness for elementary and junior high school subjects but not for senior high school students". (Lester, 1980)

Subject Variables

Defining an independent variable as one from which behaviour can be predicted, leads to a consideration of the characteristics of good problem solvers, appropriate timing of different types of problems and strategies and other subject related questions. There appears to have been only a limited number of studies which investigate developmental changes in problem solving behaviour and all of these naturally concentrated on young children. Lester's (1975) study with subjects in Grades 1 to 12 yielded results of more significance to the lower grades than to the higher ones.

One of the most notable studies on individual problem solving abilities is that of Krutetskii (1976). His main aim was "to clasify the features that characterize the mental activity of mathematically gifted pupils as they solve various mathematical problems". (p.78) Krutetskii had previously found that less able students could be grouped in six categories according to their visual and verbal abilities and he tried to apply this categorization to all students in his later study. He concluded that there appears to be three types of students - analytic, geometric and harmonic, being a combination of the first two. He also found that a distinguishing feature between good and poor problem solvers was the ability of the good problem solver to perceive the important elements of the problem. This conclusion was further substantiated by Silver (1979) who found that there was a direct relationship between a student's ability to sort problems on the basis of their underlying structure and his ability to solve problems.

There has been some criticism of Krutetskii's work on the grounds that he has not followed standard procedures of experimental design. His use of clinical investigations, however, are worth noting.

Fennema (1974) and others have investigated the differences between males and females on problem solving ability. This work is fairly inconclusive but there seems to be some general agreement that the sex-related differences increase with age so the number of years of studying mathematics may well influence the extent of the difference. When sex as a factor was controlled, the main differences between sexes lay with two affective variables, confidence in learning mathematics and mathematics as a male domain. (Fennema and Sherman, 1978)

Process Variables

Process variables include all thinking done in the solution of a problem and have been given a considerable amount of research attention. There is, however, some confusion obvious in the literature as to the meanings of the various types of processes. The term "heuristic" for instance, is to Polya (1975) a mental operation only. Others use the word more widely to mean any process a problem solver uses.

The most common method of data gathering is the "thinking aloud" technique with possible use of audiotape or videotape recording. Introspection, retrospection, and written inventories are the other techniques.

Kilpatrick (1967) devised a coding system for analysing tape recorder protocols and found the only process that had a significant correlation with the total score on a problem solving test was that of successive approximation. Also, subjects who used a trial and error process more did better than the subjects who did not use trial and error to such an extent. Webb (1975) however, found that subjects who used moderate amount of trial and error did better than those who used a substantial amount.

Krutetskii introduced clinical investigations to probe into children's mental processes. This was a significant change in methodology.

Process inventories have been designed by several researchers. One such is that of Burton et al at the Open University.

Research strongly agrees that problem solving performance is enhanced by teaching students to use a variety of strategies and heuristics both general and specific. (Suydan, 1980)

To what extent can an individual be trained to be a better problem solver? What are the conditions necessary for improvement in problem solving?

Polya's four stage model and its application to instruction has influenced the study of problem solving greatly. Hatfield (1978) suggests three types of instruction in problem solving:

1. Teaching <u>for</u> problem solving, i.e. focuses on the acquisition of concepts and skills which are needed for solving problems.

2. Teaching <u>about</u> problem solving, i.e. modelling good problem solving behaviour and emphasising certain important strategies and skills.

3. Teaching <u>via</u> problem solving i.e. presenting mathematics content in the context of problems to be solved.

Polya encourages the third type of instruction as does the Australian Mathematics Education Project.

Research studies on this aspect of problem solving can be categorised according to the methods used:

(a) Studies designed to develop specific skills such as solving equations, making tables, etc. have been concentrated at primary and lower secondary levels.

(b) Studies designed to develop specific thinking processes, appear to indicate that instruction in the use of specific problem solving behaviour such as drawing diagrams and constructing equations increases the effective use of such behaviours. (Vos, 1976) (grades 9, 10, 11)

(c) The effect of general heuristics was studied by Post and Brenman (1976) who found that instruction in a general heuristic method had no significant effect on problem solving performance. Goldberg (1974) concluded "that instruction in some heuristics of proof with reinforcement through class discussion is effective for some students". Schoenfield (1978) in a study of science and mathematics college students found that the students who were instructed on the use of heuristics such as induction, drawing a diagram, proof by contradiction, reducing the number of variables and establishing subgoals were superior to those students who did not receive the same instruction.

Ledbether (1978) tried so evaluate the effectiveness of a college level course in heuristic problem solving. She found that heuristic instruction may be more beneficial for certain ability types than for others.

Possible Trends

1. There has been a definite change of emphasis towards linking problem solvers to the types of cognitive processes they employ. Research on process variables, then, is likely to focus on creating techniques for accurately recording problem solving behaviour and on analysis of this data.

2. Investigations will continue into the interaction between the structure of the problem and problem solving behaviour.

3. Clinical studies of the effectiveness of instruction in heuristics seem to be replacing studies comparing methods of instruction in general heuristics with instruction in task-specific heuristics.

4. More emphasis will be placed on instruction - related issues.

5. A theory of mathematical problem solving appears likely to emerge. The position paper of AMEP could well be the forerunner of something more substantial.

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"IS THIS A THINKING OR AN ORDINARY ONE?" OR HOW THE NATURE OF A PROBLEM IS HELPING DEVELOP SUCCESSFUL PROBLEM SOLVERS

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In six years of research on the teaching and learning of problem solving at the Papua New Guinea University of Technology[1], the following definition of a real problem has been found useful:

> "A problem is a solution seeking situation to which automatism does not provide a ready, effective answer".

The utility of this definition results mainly from its connotations, thus:

A problem situation is a personal situation. A solution seeking situation may be a problem to one person but not to another. It is not a problem to one who readily gets its answer because he sees its equivalence to a situation he has met and solved earlier. On the other hand it is a problem to one who cannot get its answer without resorting to productive and creative thinking which consists of repatterning and restructuring past experiences to meet the new situation. This personal nature of a problem connotes two characteristics of a problem — its novelty and difficulty.

At this stage the research is in the form of teaching experiments to find out if

1. developing an awareness and appreciation of the nature of a problem,

2. providing the means and the opportunity to solve problems in groups which simulate the discussions of a village forum,

3. teaching an algorithm for solving a problem,

will produce successful and efficient problem solvers.

Hence the first and most important and also most difficult task of the researcher was the selection of solution seeking situations which are problems to the subjects of the study.

The difficulty of the task stems from the properties of novelty and difficulty of a problem.

The solution seeking situation must be new to the subjects of the teaching experiment. The solution seeking situation must be difficult, yet not too difficult that the subjects will not desire its solution. Hence the choice of problems had to be done only after a very careful scrutiny of the students past experiences, their range of mathematical and linguistic abilities; their store of previously acquired mathematical knowledge, their levels of interests.

This year, 1983, the subjects of the teaching experiment are students enrolled in the preliminary year of Surveying Mathematics. There are 18 students in the class. Their level of achievement in mathematics and of maturity are shown in Table 1.

MATHEMATICAL ACHIEVEMENT → / MATURITY* ↓	UPPER PASS	CREDIT	DISTINCTION	TOTAL
School Leavers	1	5	3	9
Non-school Leavers	3	4	2	9
Total	4	9	5	18

TABLE 1: Mathematical Achievement and Maturity Levels of the Eighteen Students Enrolled in Surveying Mathematics for the Preliminary Year in 1983 at the Papua New Guinea University of Technology

(*The school leavers are students who came directly from Grade 10 of the Provincial High Schools in 1982. Non school leavers are students who worked and/or attended technical colleges or earned certificates in Surveying after coming from the Provincial High Schools, hence were more mature than the school leavers).

This year the teaching experiment had a new feature. In addition to asking the subjects to solve problems in structured group sessions (which are being videotaped), they were given more individual practice in problem solving by asking them to do unit assignments. These assignments were incorporated into the course syllabus (and hence also into its marking scheme) as follows:

In the preliminary year the students are taught mathematics by self-paced methods. In this method the learning material is presented as a series of core units which the student works through and passes on his own time. This year, as an added requirement to a pass on a core unit the students are required to answer an assignment which they also worked out on their own time, but which they handed in before sitting for a Unit Test. The questions in the assignment were based on the unit but involve cognitive levels higher than mere recall

of knowledge. One question in particular was a problem to the students.

Questions and discussions to develop the appreciation and awareness of the nature of a problem and the 4 steps of Polya's problem solving algorithm[2] were marked and given back to the students, starting with the assignment for the first unit, Unit 102. Thus the students worked out the problem for Unit 102 without the treatment. The problem was:

> "You might like to draw a number line to help you solve this one. If it takes 15 seconds to strike 6 o'clock, how long does it take to strike 12 o'clock?"

From the many requests for clarification of this question, it was evident that this could be a problem to the subjects. That all 18 subjects attempted the solution could indicate their desire to solve it (although the motivation for the desire could be negative, e.g. fear of getting low marks, it was the first assignment, etc.). No one gave the correct answer however. Could this be because the problem was too difficult or because they were not yet aware of the nature of a problem or because they were not explicitly taught how a problem could be solved?

After this initial disappointment the number of successful problem solvers gradually increased. Could this be because most verbal interaction in the classroom (whether teacher to whole class, or teacher to individual student) always

(a) emphasized the nature of a problem as a situation which is new and therefore difficult and requires much thinking, and

(b) pointed out the usefulness of the steps in Polya's problem solving algorithm,
e.g.

"What do you know?"

"What is needed?"

"What will you do to find the unknown from the given?"

"How will you know if your answer is right?",

The number of successful problem solvers in the assignments are given in Table 2.

CORE UNIT	NUMBER OF SUCCESSFUL PROBLEM SOLVERS	%
102	0	0%
103	4	22%
106	0	0%
107	7	38%
104	9	50%
112	5	27%
109	9	50%
110	9	50%
111	10	55%
114	9	50%

TABLE 2: Number of Successful Problem Solvers In the Assignments for Core Units in MA 150 in the first Semester of 1983

*All 18 students attempted the problem for each of the 10 assignments. except for the assignment for Unit 114 which was attempted by all except by one student who wrote "not sure".

**The "Problems" are given at the end of the paper.

At this stage, aside from cautiously observing the increase in the number of successful problem solvers and hoping the trend will continue in the second semester, many questions still have to be answered before conclusions can be made. A big question, one which is anticipated to limit the validity of the results of the research is that of whether a chosen "problem" is a real problem to each of the subjects of the experiment. Were the problem really new and therefore difficult, yet capable of solution? Do their solution require thinking and not mere recall of algorithms, processes, etc. As one of the students so aptly wondered:

"Is this a thinking or an ordinary one?"

"Problems" in the Assignments for the Ten First Semester Core Units

Unit 102. You might like to draw a number line to help you solve this one: If it takes 15 seconds to strike 6 o'clock, how long does it take to strike 12 o'clock?

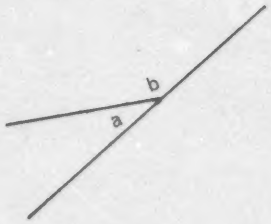
Unit 103. I decide to make a file of paper. The paper I use is $\frac{1}{50}$ cm think. I first put down a single sheet. Then I double what I have put down making 2 sheets. A second doubling gives me 4 sheets. A third doubling makes 8 sheets. If I double the number of sheets 10 times in all, how high is the file?

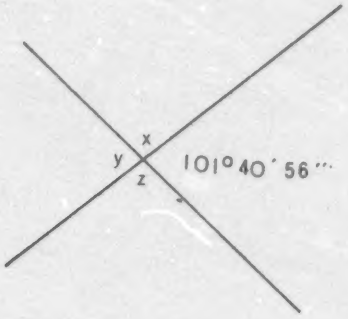
Unit 104. In a certain town in Papua New Guinea, ten and a half mm of rain fell in a week and a half. What was the average daily rainfall over the period?

Unit 106. A PMV leaves Lae for Goroka each hour on the hour, and an identical arrangement applies for PMV's from Goroka to Lae. All PMV's travel at the same speed. If I travel from Lae to Goroka, how many PMV's do I pass if the journey takes $4\frac{1}{2}$ hours?

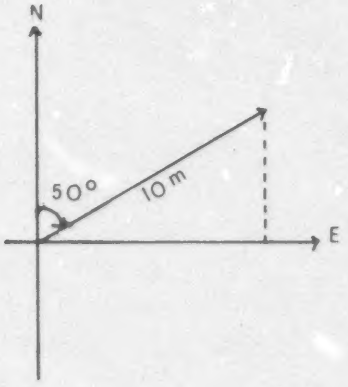
Unit 107. Find x if $\frac{x^2 + x}{x + 1} = 2$

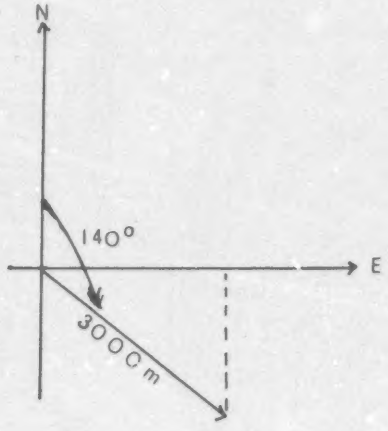
Unit 109. The sum of the angles a and b in the figure on the left is 180°. Find x, y, and z in the figure on the right.





Unit 110. Find x in each of the followi g by using trigonometry. The figures are not drawn to scale.





Unit 111. A boat has a velocity in still water of 8 km/hr. The river current has a velocity of 3 km/hr westward.

(a) What is the resultant velocity of the boat eastward?

(b) What is the resultant velocity of the boat westward?

(c) What is the resultant velocity of the boat northward?

Unit 112. The sides of a square are doubled. What happens to its area?

Unit 114. Express 1 000 000 000 as the product of two number each containing zeros.

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DUBE, LILIA.S. "Research on the Teaching of Problem Solving at the Papua New Guinea University of Technology: Retrospect and Prospect". In *Research in Mathematics Education in Papua New Guinea, 1981*, ed. by P. Clarkson, Mathematics Education Centre, Papua New Guinea University of Technology, Lae, 1981, pp. 38–42.

POLYA, G. *How to Solve It*. Princeton University Press, Princeton, New Jersey, 1967, pp. xvi–xvii.

A NOTE ON PROBLEM SOLVING

KATHLEEN COLLARD
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One aspect of problem solving is the reaction of the student in being faced with a problem. The student may meet the problem as a challenge, and this is likely to be the most fruitful approach. In many cases and here in Papua New Guinea it appears to be the rule rather than exception, students do not have sufficient confidence in themselves and appear to be frozen into inaction when faced with a problem.

The first task then, in teaching a student to solve problems is, by some means or other, to build up the student's confidence in himself. The main difficulty is usually not in doing the mathematics of the problem but in translating the problem into mathematical language and in deciding what is the mathematics that has to be done. The student should be given practice in this, beginning with elementary situations in which he can hope to succeed, it being recognized that nothing encourages confidence like success.

TOWARDS LOCALIZATION

A.M. ROBERTS

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1. BACKGROUND

Both U.P.N.G. and Unitech are experiencing difficulties in obtaining Nationals who are interested in a career in University mathematics teaching, and then in training those that do show interest up to a level where they can succeed in postgraduate studies overseas. This paper analyses the skills required for postgraduate mathematical studies and then makes suggestions as to how these skills may be better developed in tertiary maths courses in this country.

Research degrees in Mathematics essentially consist of the solving of a large new problem, or the establishment of proof of a new theorem, or the development of complicated computer programs, or a combination of these.

Coursework Masters or Honours programs are a preparation for this. There is an emphasis on proofs – the <u>why</u> rather than the how of solving problems. The techniques are assumed known. Problems get fewer but longer the more advanced the coursework becomes. Gaps in knowledge of techniques are assumed to be filled in by outside reading.

The problem solving requirements of advanced mathematical studies may be summarized as follows:

1. Knowledge of techniques.
2. Ability to understand proofs (and model real life situations).
3. Ability to do long problems.
4. Ability to translate language into formulae.

We now consider how Papua New Guinean graduates match up to these requirements.

2. CURRENT P.N.G. ACHIEVEMENT

2.1 Mathematical courses in Papua New Guinea cover most of the skills required for advanced work, at least superficially. Graduates may not always quickly recall techniques, perhaps because of the superficial treatment given. Nevertheless there doesn't appear to be too much difficulty in following routine mathematical techniques after some revision.

2.2 Proofs of results are avoided, or given in simplified (non rigorous) form, without being made examination material in Papua New Guinea mathematics courses. Special cases or exceptions to usual results are not considered on the grounds that they confuse students. However, it is these exceptional situations which give rise to quite a lot of advanced mathematics. Modelling and developing new results depends heavily on the logical ability of the research worker. Overseas graduate mathematics courses consist of more than 50% proofs or theory, and it is here that Papua New Guinea graduates experience extreme difficulty.

2.3 Mathematics courses in Papua New Guinea stress problems that the average student can do, and test specific areas of knowledge through short questions on a single feature rather than synthesis of several techniques. A good mathematician must hold the overall goal in mind while working through various techniques in the detail of his work.

2.4 Translation of words into symbols is harder in a second language than it is to a native speaker, but for a graduate specializing in a field of his own choice, this should not be too much of a problem. There is, perhaps, insufficient opportunity for Papua New Guinean mathematicians to verbalise their mathematics through seminars or essays during their undergraduate courses.

3. EVIDENCE

In setting examinations in Papua New Guinea, long questions are avoided, or if a lengthy question is set, it is divided into specific subsections, each requiring limited technical skills. Longer assignment problems are rarely completed correctly due to minor blunders in the working of better students, or gross misunderstandings of weaker students.

Because of the slow pace of working of many students, a minimum of problems are set and routine questions are preferred to those which extend topics beyond the confines of the syllabus. I was disappointed when a group of post-graduate students at Unitech completed on average two thirds of a set of problems I would have set for first year undergraduates in Australia. Even the more thorough of the students failed to get to grips with the parts of questions which tried to extend the work away from the mainstream syllabus. I was not surprised when they lacked the routine skills to synthesize the work into more complicated tasks at the end of the course.

The first course involving proofs at University of Papua New Guinea was the third year unit Introduction to Pure Mathematics. This course was found extremely difficult by most students, and a grade above D was rare. One year, in preparation for this course (and as background for trainee teachers), I taught some Euclidean Geometry in a second year unit (the course content was changed

the following year). It was the first time students had to decide what was known, what was to be proved, and then use logical steps to obtain the result. The better students adapted to this in the four weeks of the geometry segment, but weaker students (at least half the class) were unable to understand what was going on. This shows that proofs can be successfully taught if there is time available. By continually avoiding theory, Papua New Guinean graduates are badly disadvantaged in tackling further mathematics overseas.

In my undergraduate studies (and even at the end of Grade 12), exams consisted of long questions, not all of which had to be completed, but 'more credit will be given for complete answers than for an equivalent quantity of fragments'. This encouraged students to aim at integrating their knowledge into solution of multi step problems, rather than learning each technique in isolation. It also encouraged accurate work and checking at each step to ensure a complete answer.

I recall first becoming aware of this synthesis of several topics in co-ordinate geometry where, for example, calculus is used to get the slope of a curve, the equation of a tangent can then be found, and finally simultaneous linear equations are used to find points of intersection of the tangent with other lines. Another area which is available at Unitech for engineering students is Newtonian dynamics using trigonometry for components of forces, calculus for equations of motion, and with simultaneous linear equations commonly occurring.

I have tried to teach two units adapted from third year undergraduate work in Australia to postgraduate students in Papua New Guinea (one at U.P.N.G. and one at Unitech). Both emphasised lecture material (theory) over exercises, and were in applied areas (tensors and fluid mechanics). The results were very disappointing. Students here didn't show any understanding of the theory taught. Some basic

definitions were learned, but without much understanding. Students were unable to use techniques which were well within their capabilities, to solve simple (but multiple step) fluid problems, even when similar problems had been discussed in class.

4. RECOMMENDATIONS

The areas of proofs and solutions of multi step problems appear to be weaknesses in Papua New Guinean mathematics graduates. To remedy this, more proofs should be introduced into undergraduate courses as examinable material. An introductory course in Euclidean Geometry (which combines visual diagrammatic work with formal logic), or some other area where logic can be emphasised should be taught at second or third year level for all those students who show mathematical aptitude.

The tendency toward short single technique questions on exams and assignments should be reversed to encourage synthesis of different techniques into longer, more realistic, problem situations. Topics which lend themselves to such applications are:

(i) Proof of the quadratic formula (the first long algebraic manipulation encountered in secondary maths).

(ii) Trigonometric Indentities (selection of which trig formulae to use, and keeping an eye on the final result while doing the detailed manipulation).

(iii) Co—ordinate Geometry (conics, tangents, normals, parametric equations, calculus).

(iv) Mechanics.

(v) Euclidean Geometry (proofs and corollaries).

All these topics could be included in U.P.N.G. or Unitech courses. The reason for omitting trig identities and longer co-ordinate geometry problems is shortage of time, when the results are of no immediate use. The effect of this omission is to produce graduates who are unable to follow through longer calculations. The omission of proofs reduces problem solving ability by not developing capacity for logical deduction of the consequences of what is known and thus finding new (perhaps useful) information.

DECOMPARTMENTALISATION

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One obstacle to problem solving, and often to finding good approaches to questions not dignified by the title "problem", is our inclination to keep our knowledge in separate compartments. While this may have some useful role – we could not cope with all our knowledge being activated at once – it would seem that we often limit access to relevant compartments on the basis of criteria unconnected with the matter in hand. In the United Kingdom the author noted, over a period, a number of examples he encountered or heard about. These mainly concern not "problems" but "straight" mathematics. But any successes in overcoming compartmentalisation in this area would be a help in problems, where indeed it is more crucial.

Among the most striking of the examples noted were:–

> in the course of a test on routine calculations, 1st year university students (Mechanical Engineering), meeting the absurdly simple $3 \times (2 \div 3)$, mainly answered in decimals, usually somewhat incorrectly e.g. 1.998 or 2.001. (Aids available were tables, not calculators).
>
> 1st year university students (Electrical Engineering), having performed competently in complex numbers in classes and assignments given by the Mathematics Department (up to e.g. finding real and imaginary parts of $\tan(x + jy)$), were subsequently alleged by an Electrical Engineering Lecturer to know next to nothing about complex numbers.
>
> 1st year university students (in Mathematics) who, in an assignment for one course of lectures, had successfully found roots by de Moivre's Theorem, less than a week later stopped short at $m = -2$ in treating the auxiliary equation $m^3 = -8$ arising from a 3rd order differential equation in an assignment associated with a different course of lectures.

The last two of these have something in common with the blocks due to compartmentalisation in problems proper — an inability or unwillingness to deploy knowledge in unfamiliar surroundings. Here different "surroundings" might refer to contexts, courses, or even different lecturers.

Other instances show us trying to work at a level of sophistication associated with an earlier period of development rather than use updated information. We can distinguish two kinds of compartmentalisation here:—

(a) A kind of infantile regression. For instance:

> A student goes from $4\sin^3\theta = \sin\theta$ to $4\sin^2\theta = 1$ in the course of solving $\sin 3\theta = 2\sin\theta$.

The student referred to would never infer $x = 4$ from $x^2 = 4x$ and would probably not infer $4\sin^2\theta = 1$ from $4\sin^3\theta = \sin\theta$ if the latter was the question given; however he actually reaches this equation through the relatively arduous task of using a formula for $\sin 3\theta$ and, either because of the consequential stress or through relaxing attention after the "hard" part, he fails to bring his knowledge about not dividing by zero into play — such knowledge was of course non—existent or not needed when he first met dividing equations, that is, in examples like $3x = 6$.

(b) A kind of sales resistance. We don't bother to assimilate a new method because we already know one method — the compartment is, as it were, full up, e.g.

> A student integrates $1/x(x + 2)(2x - 1)$ taking a page or two to find the partial fractions instead of using a quicker way he met later. With luck he still gets the right answer, in contrast to his fellow student who, for similar reasons, failed to digest the new method but then, through stress (or laziness), displayed syndrome (a) and wrote $\log [x(x + 2)(2x - 1)]$ for his answer.

Resistance to learning a new way to do something we are already able to do one way is a familiar enough phenomenon in other fields – the self-taught sportsman or pianist may not take kindly to advice on how to do better, even when he is paying for it. In the area of problem-solving it may limit people who at one stage were good problem-solvers. In particular a proportion of those who were relatively good, before meeting algebra, at solving problems arithmetically, resist, understandably but ultimately to their cost, the efforts of teachers to introduce them to the art of translating problems into algebraic form.

How can teachers help to overcome compartmentalisation?

1. Merely pointing out when it is occurring may help. If, through experience or futher quizzing, one can offer a correct explanation of why it is occurring so much the better. However, to rest too many hopes on this would be to commit what might be called the psycho-analytic fallacy. But one should be able to offset any depression which might be caused in confirming that the student has a weakness he half knows about but would prefer to ignore, by pointing out areas (not necessarily in mathematics) in which the teacher has been known to display a similar weakness.

2. Encouragement to read around the subject. The "right" book may vary from a text-book which treats the course material a little differently, to wider ranging books like, say, those of Edna Kramer or W.W. Sawyer, or Readings from the Scientific American. It will obviously depend on level, but possibly also on a student's other interests. Unitech's library can cater for most variations if students can be persuaded to browse in it. A short recommended list is at once a near-impossibility, because of the vast and arbitrary exclusions

involved, and a necessity for many students if they are to take the plunge of starting to use a library in this way.

3. Telling students to be on the look-out for shorter or neater ways of doing things. Neater ways often show students' knowledge being used less routinely, and uncover unsuspected connections – so searching for them, though it may consume a little more time now, will save a lot later. (Also it can be fun).

   Nevertheless we should recognise that most students will but rarely make our problems their own in this or similar ways, such as making up their own questions from facets of ours – something one is liable to meet in the area of computing, but which is also illustrated in the first example below.

4. By judicious mention, where apposite, of students' — success stories in the field of decompartmentalisation.

   Without examples of such success this paper would better have been entitled compartmentalisation than DE – compartmentalisation.

1st example: A student knew the standard integrals of $1/(1 + x^2)$ and $1/\sqrt{1 - x^2}$. He could remember that the integrals of $1/(a^2 + x^2)$ and $1/\sqrt{a^2 - x^2}$ were constants times $\tan^{-1}(x/a)$ and $\sin^{-1}(x/a)$, but in applications frequently confused which constant was which. He also felt that checking by differentiation, or integrating by substitution, was too tedious to bother with every time. He resolved this dilemma by recourse to the method of dimensions he had met in Mechanics and Physics! Evidently x^2 and a^2 had the same dimensions, so suppose x and a were both lengths. Then dx (or at any rate δx before passage to the limit) must

be construed as a length. Thus

$\int \frac{dx}{a^2 + x^2}$ has dimensions $\frac{L}{L^2} = L^{-1}$, while

$\int \frac{dx}{\sqrt{a^2 - x^2}}$ has dimensions $\frac{L}{\sqrt{L^2}}$ = pure number.

Hence the constants to multiply $\tan^{-1}(x/a)$ and $\sin^{-1}(x/a)$ must be $1/a$ and 1 respectively.

More trouble to think out in the first instance than conventional processes; but how much more unforgettable than any mnemonic, once found.

2nd example: An exercise on integration by parts was to integrate $x \ln(x + 4)$. Most students got $\frac{1}{2}x^2 \ln(x + 4) - \frac{1}{2}\int \frac{x^2}{x + 4}\, dx$ and had varying fortunes with the new integral. But one wrote $\frac{1}{2}(x^2 + C)\ln(x + 4) - \frac{1}{2}\int \frac{x^2 + C}{x + 4}\, dx$, and then chose -16 for C to make the new integral simplify dramatically.

In contrast to the 1st example the knowledge for producing a neat solution is here taken from a near rather than a distant compartment. Nearness notwithstanding, and despite effectiveness in finding indefinite integrals of, say, $\ln(x + 4)$, $x \ln(x^2 + 3)$ and $\tan^{-1} x$, this ingenious notion of setting the arbitrary constant to work for us instead of being a necessary evil (the usual student view), scarcely appears in books except perhaps in connection with definite integrals where it is sometimes used to give zero at one of the limits.

There would seem to be a case for building up a compendium of students' triumphs in decompartmentalisation at various levels, not only at university but also in schools. The foreword to "The Surprise Attack in Mathematical Problems"*, a book based on problems contributed to a puzzle column,

says that in nearly all cases the prize-winning solutions by readers excelled those of the contributors. We do not expect this to hold at Unitech or in schools. But it may be that teachers, especially inexperienced ones, learn more from their students in this way than they would care to admit.

* L.A. Graham. Dover Publications Inc., 1968.

UNIVERSITY OF TECHNOLOGY

MATHEMATICS EDUCATION CENTRE

THE THIRD NATIONAL

MATHEMATICS EDUCATION CONFERENCE

5–6 MAY 1983

VENUE – Audio Visual Theatre, University Library

FORMAT – Each presentation will be of half an hour. A paper will be talked to for approximately twenty minutes leaving ten minutes for discussion.

PROGRAM

THURSDAY – 5TH MAY

8.05 – 8.30 Opening and Welcome
Professor L.J. Lewis – Vice Chancellor

Session 1 – Microcomputers and Education
Chairman – P.C. Clarkson

8.30 – 9.00 'Instruction on Computer – Capability'
Mr. H. Fredericks – Goroka Teachers' College

9.00 – 9.30 'Limitations of the Role of Computers in Education'
Dr. C. Wilkins – Mathematics Department

9.30 – 10.00 'The Use of Computers for Concept Development'
Mr. R. Seldon – Mathematics Department

10.00 – 10.30 MORNING TEA

Session 2 – Reports on Research
Chairman – P. Sullivan

10.30 – 11.00 'The Training of Spatial Abilities and its Relation to Mathematics Education: A Review
Mr. G. Lean – Mathematics Department

11.00 – 11.30 'A Case Study on Drawing Nets of Solids'
Mr. J. Gough – University of Papua New Guinea

11.30 – 12.00 'Problem Solving: Adapted for Yapese Elementary Schools'
Mr. D. Nelson – Ministry of Education, Yap, Federated State of Micronesia

12.00 – 1.00 LUNCH

Session 3 – Post Year 10 Mathematics
Chairman – R. Seldon

1.00 – 1.30 'Monitoring Standards in Mathematics of the Grade 10 Output'
Mr. L. Ros – Goroka Teachers' College

1.30 – 2.00 'Decision-making and the National High School Curriculum'
Mr. J. Bailey – Ministry of Education

2.00 – 2.30 AFTERNOON TEA

Session 4 – University Mathematics
Chairman – N. Roberts

2.30 – 3.00 'Analysis of 1st Year UPNG Mathematics Examination Performance'
Mr. J. Gough – University of Papua New Guinea

3.00 – 3.30 'Mathematics – A Preparation for Study in Engineering and Applied Science'
Mr. M. Priest – Electrical Engineering Dept.

3.30 – 4.00 'Engineering Mathematics – A Problem of Logic'
A/Prof. Snell – Civil Engineering Dept.

FRIDAY – 6TH MAY

Session 5 – Mathematics Education Centre
Chairman – P.C. Clarkson

8.15 – 8.45 'The Mathematics Education Centre – An Overview'
Mr. P.C. Clarkson – Mathematics Education Centre

8.45 – 9.15 'Proportionality Skills of Grade 10 School Leavers'
Mr. P. Sullivan – Mathematics Education Centre

9.15 – 10.00 'A Progress Report on Research in Progress'
Dr. E. Southwell – Nepean College of Education, New South Wales, Australia

10.00 – 10.30 MORNING TEA

Session 6 – Provincial High School Mathematics
Chairman – P.C. Clarkson

10.30 – 11.00 'From Arithmetic to Algebra'
Mr. M. Britt – Auckland Teachers' College, New Zealand

11.00 – 11.30 'Professional Development of Mathematics Teachers: Experience of the Math Groups'
Mr. Robert Maioni – Ministry of Education

11.30 – 12.00 'Availability of Mathematics Resources in Morobe P.H.S.: A Preliminary Report'
Mr. A. Koigiri – Mathematics Education Centre

12.00 – 1.00 LUNCH

Session 7 – Problem Solving at Unitech
Chairpersons – T. Fairclough/H. Whippy

1.00 – 1.30 'A Review of the Problem Solving Literature'
Dr. E. Southwell – Nepean College of Education, New South Wales, Australia

1.30 – 2.00 'Problem Solving in Geometry at Unitech'
Dr. L. Dube – Mathematics Department

2.00 – 2.15 Mr. B. Landau

2.15 – 2.30 Mr. G. Lean

2.30 – 2.45 Professor K. Collard

2.45 – 3.00 Dr. N. Roberts

} Views from Mathematics Department staff

3.00 – 3.15 AFTERNOON TEA

Forum on Problem Solving at Unitech
Chairperson – H. Whippy

3.15

The following M.E.C. Reports have been published:

No. 1: A. EDWARD 'Report for 1978, including a Short History of the Mathematics Learning Project and Mathematics Education Centre up to 1977', 1978.

No. 2: A.J. BISHOP 'Spatial Abilities in a Papua New Guinea Context', 1978. (out of print)

No. 3: C. MEEK and N. FERIL 'A Pilot Study into the Level of Comprehension of Some Words taken from the Papua New Guinea Community Schools Mathematics Syllabus', 1978.

No. 4: A.J. BISHOP 'Developing Spatial Abilities', 1977.

No. 5: A. EDWARDS 'A Study of the Issues Involved in Establishing the Mathematics Education Centre and a Report on its Work from December 1978 to May 1979', 1979. (out of print)

No. 6: A. EDWARDS and A. BAJPAI 'Preliminary Report on the Basic Arithmetic Test carried out in Post-Secondary Institutions in Papua New Guinea in 1979', 1979 (out of print).

No. 7: A. EDWARDS 'Final Report on the Establishment of the Mathematics Education Centre', 1979 (out of print).

No. 8: G.A. LEAN 'The Training of Spatial Abilities: A Bibliograph', 1980.

No. 9: G.A. LEAN 'An Investigation of Spatial Ability Among Papua New Guinea Students', 1980.

No. 10: G.A. LEAN and M.A. CLEMENTS 'Spatial Ability, Visual Imagery and Mathematical Performance', 1980.

No. 11: P.L. JONES 'The Introductory Mathematics Programme at the Papua New Guinea University of Technology - A Survey of Student Opinion', 1980. (out of print)

No. 12: P.L. JONES 'A Study of Proportional Reasoning', 1980. (out of print)

No. 13: M.A. CLEMENTS and G.A. LEAN 'Influences on Mathematical Learning in Papua New Guinea: Some Cross - Cultural Perspectives', 1981, (2nd edition). (out of print)

No. 14: M.A. CLEMENTS 'Careless Errors Made by Sixth-Grade Children on Written Mathematical Tasks', 1981 (2nd edition).

No. 15: P. SULLIVAN 'The Standards Debate: An Alternate Strategy', 1981. (out of print)

No. 16: P.L. JONES 'Mathematical Language and the Learning of Mathematics in Papua New Guinea', 1981. (out of print)

No. 17: F. AUSBURN and L. AUSBURN 'Perception, Imagery and Education in Developing Countries', 1981. (out of print)

No. 18: M.A. CLEMENTS and P.L. JONES 'The Education of Atawe', 1981.

No. 19: P.L. JONES 'Learning Mathematics in a Second Language: A Problem with More and Less', 1981.

No. 20: M. SIFORD 'A Study of Algebra in the Provincial High Schools Curriculum', 1981.

No. 21: F. AUSBURN and L. AUSBURN 'Cognitive Styles and Visual Skills in Papua New Guinea Students', 1982.

No. 22: P.C. CLARKSON 'Mathematics Learning in PNG National High Schools: The Analytic – Visual Variable', 1982.

No. 23: P.C. CLARKSON 'Mathematics Learning in PNG National High Schools: A Comparison of Years 11 and 12 Across Schools', 1982. (out of print)

No. 24: P. SULLIVAN 'Practical Aspects of Drug Calculations', 1982.

No. 25: P.C. CLARKSON 'Year 12 Students' Mathematical Performance on the 1980 and 1981 External Examinations', 1983.

No. 26: P.C. CLARKSON 'Types of Errors Made by Papua New Guinean Students', 1983.

No. 27: P.C. CLARKSON 'Papua New Guinean Students' Causal Attributions for Success and Failure in Mathematics', 1982.

No. 28: P. SULLIVAN 'Remediation of Basic Arithmetic Weaknesses in Post-Secondary Students', 1983.

*Those reports indicated as 'out of print' should be available from the Matheson Library, Papua New Guinea University of Technology on inter-library loan.

The following M.E.C. Technical Reports have been published:

No. 1: P. SULLIVAN 'Report of the First Conference to Prepare Mathematics Material for Post-Secondary Courses', 1981. (out of print)

No. 2: P.C. CLARKSON 'Report of the 1980 National High School Year 12 Mathematics Marking Panel', 1981.

No. 3: 'Annual Report, 1980', 1981.

No. 4: 'Annual Report, 1981', 1982. (out of print)

No. 5: P.C. CLARKSON 'Report on the 1981 National High School Year 12 Mathematics Examinations', 1981. (out of print)

No. 6: P.C. CLARKSON 'Pre-University Mathematics Course', 1982.

No. 7: P. SULLIVAN 'Report of Second Program to Prepare Mathematics Materials for Post-Secondary Courses', 1982. (out of print)

No 8: P.C. CLARKSON 'Report on the 1982 National High School Year 12 Mathematics Examinations', 1983.

No. 9: 'Annual Report, 1982', 1983.

No. 10: 'Problem Solving through the Curriculum', 1983.

No. 11: P. SULLIVAN 'Report of the Third Program to Prepare Mathematics Materials for Post-Secondary Courses', 1983.

Other publications include:

P. SULLIVAN 'An Introduction to Nursing Mathematics', 1981. (out of print)

P.C. CLARKSON (ed) 'Research in Mathematics Education in Papua New Guinea, 1981', Proceeding of the First National Conference on Mathematics Education, May, 1981. (out of print)

P.C. CLARKSON and P. SULLIVAN 'Vocational Mathematics', 1982.

P.C. CLARKSON (ed) 'Research in Mathematics Education in Papua New Guinea, 1982', Proceedings of the Second National Conference on Mathematics Education, May, 1982.

*Those reports indicated as 'out of print' should be available from the Matheson Library, Papua New Guinea University of Technology on inter-library loan.